Unification and Resolution

Exercises

1 Substitutions

Exercise 1. Let

$$\alpha = \{\ x/a,\ y/f(b),\ z/c\ \} \qquad \beta = \{\ v/f(f(a)),\ z/x,\ x/g(y)\ \}$$

- compute $\alpha\beta$ and $\beta\alpha$
- for every of the following formulæ, compute (i) $F\alpha$; (ii) $F\beta$; (iii) $F\alpha\beta$; (iv) $F\beta\alpha$
  1. $p(x, y, z)$;
  2. $p(h(v)) \vee \neg q(z, x)$
  3. $q(x, z, v) \vee \neg q(g(y), x, f(f(a)))$
- are α, β, $\alpha\beta$ or $\beta\alpha$ idempotent?

2 Unifiers

Exercise 2. For every C_1, C_2 and α, decide whether (i) α is a unifier of C_1 and C_2; and (ii) α is the *MGU* of C_1 and C_2.

C_1	C_2	α
$p(a, f(y), z)$	$q(x, f(f(v)), b)$	$\{\ x/a,\ y/f(b),\ z/b\ \}$
$q(x, h(a, z), f(x))$	$q(g(g(v)), y, f(w))$	$\{\ x/g(g(v)),\ y/h(a, z),\ w/x\ \}$
$q(x, h(a, z), f(x))$	$q(g(g(v)), y, f(w))$	$\{\ x/g(g(v)),\ y/h(a, z),\ w/g(g(v))\ \}$
$r(f(x), g(y))$	$r(z, g(v))$	$\{\ x/a,\ z/f(a),\ y/v\ \}$

3 Unification Algorithm

Exercise 3. Find, when possible, the *MGU* of the following pairs of clauses.

$\{q(a),\ q(b)\}$
$\{q(a, x),\ q(a, a)\}$
$\{q(a, x, f(x)),\ q(a, y, y,)\}$
$\{q(x, y, z),\ q(u, h(v, v), u)\}$
$\{p(x_1, g(x_1), x_2, h(x_1, x_2), x_3, k(x_1, x_2, x_3)),\ p(y_1, y_2, e(y_2), y_3, f(y_2, y_3), y_4)\}$

4 Resolution with Unification

Exercise 4. Determine whether the following clauses can be factorized, and give the factors if possible.

1. $p(x) \lor q(y) \lor p(f(x))$
2. $p(x) \lor p(a) \lor q(f(x)) \lor q(f(a))$
3. $p(x, y) \lor p(a, f(a))$
4. $p(a) \lor p(b) \lor p(x)$
5. $p(x) \lor p(f(y)) \lor q(x, y)$

Exercise 5. Find the possible resolvents of the following pairs of clauses.

C	D
$\neg p(x) \lor q(x, b)$	$p(a) \lor q(a, b)$
$\neg p(x) \lor q(x, x)$	$\neg q(a, f(a))$
$\neg p(x, y, u) \lor \neg p(y, z, v) \lor \neg p(x, v, w) \lor p(u, z, w)$	$p(g(x, y), x, y)$
$\neg p(v, z, v) \lor p(w, z, w)$	$p(w, h(x, x), w)$

Exercise 6. The proof of the lemma appearing in slide 19 (page 19 of 07unification.pdf file) contains a flaw. Find it.

Exercise 7. Apply resolution (with refutation) to prove that the following formula

$$5 \quad m(5, f(7, f(5, f(1, 0))))$$

is a consequence of the set

1 $\neg m(x, 0)$
2 $\neg i(x, y, z) \lor m(x, z)$
3 $\neg m(x, z) \lor \neg i(v, z, y) \lor m(x, y)$
4 $i(x, y, f(x, y))$

Note that $f(4, f(3, f(2, f(1, 0))))$ can be seen as the list $[4, 3, 2, 1, 0]$, so that m somehow represents list membership.

Hint: remember to rename variables when necessary.

Exercise 8. Try to figure out *precise* and *simple* rules for eliminating from derivations (obtained by saturation) which are not relevant to the satisfiability of the original set.