Computational Logic

Recall of First-Order Logic

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What you ABSOLUTELY have to get

Names, individuals, true notions

Names are only there to denote things

- there is no problem in taking

  female () or even *odd*(4)

  to be true if we are always consistent about it
- *never* forget that you are dealing with symbols!

What you ABSOLUTELY have to get

Types in first-order logic!

- a function of arity n will *never* have arity $m \neq n$ in the same system
- a predicate of arity n will *never* have arity $m \neq n$ in the same system
- a function is *not* a predicate (*)
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Equality

- the predicate $=/2$ is special in the sense that its meaning is often given for granted in a formal system (i.e., being equal means being the same element of the domain)
- anyway, we could choose to redefine it!

What you ABSOLUTELY have to get

Formalization, truth and well-formedness

- a formula can be well-formed without being true
- a formula can be a correct formalization of a sentence without being true or even (to our intuition) reasonable
- don't worry: sometimes a big formula may be needed to express a small sentence, or the other way around

Interpretations

When we want to prove that a formula is not valid, we need *one* interpretation which makes it false

- you can choose *anything* you want as D and I
- it's ok if we take *that* function $s/1$ to map 45 to  !

What you ABSOLUTELY have to get

Implication

- that the left-hand side of an implication is false is enough to say that the implication is true
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What you ABSOLUTELY have to get

Implication, cont.

- *all red apples are good* $\rightsquigarrow$

$$\forall x((apple(x) \land red(x)) \rightarrow good(x))$$

  - there are apples which are red and good $\rightsquigarrow$ NO
  - all apples are red and good $\rightsquigarrow$ NO
  - there are no apples which are red but not good $\rightsquigarrow$ YES
  - the set of good apples is a superset of the set of red apples $\rightsquigarrow$ YES
  - whenever an apple is not good, it cannot be red $\rightsquigarrow$ YES
  - whenever an apple is not good, it must be red $\rightsquigarrow$ NO

- *there exists a yellow apple which is bad* $\rightsquigarrow$

$$\exists x(apple(x) \land yellow(x) \land bad(x))$$

  - whenever an apple is yellow, it is bad $\rightsquigarrow$ NO
  - there is at least an object which is bad and yellow, and is an apple $\rightsquigarrow$ YES
  - every time an apple is good, it is not yellow $\rightsquigarrow$ NO