Computational Logic

SLD resolution

Damiano Zanardini

UPM European Master in Computational Logic (EMCL)
School of Computer Science
Technical University of Madrid
damiano@fi.upm.es

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Introduction

SLD: Selection function in Linear resolution for Definite clauses

- combines linear, input, directed and ordered strategies on a particular class of clauses

Horn clauses

- at most one non-negated literal (if it exists, it's the first in the clause)
  - $A \vee \neg B_1 \vee \neg B_2$
  - A
  - $\neg B_1 \vee \neg B_2$
- clauses *without* the non-negated literal form the *goal set*
- clauses *with* the non-negated literal form the *support set*

Introduction

Definition (SLD resolution)

An SLD derivation of C_m from a set $\{C_1, .., C_n\}$ of Horn clauses (with the non-negated literal in the first place, if it exists) is a sequence $\langle C_1, .., C_i, .., C_n, C_{n+1}, .., C_m \rangle$ such that

- *C_{n+1} is the resolvent of C_i (goal clause) and another $C \in \{C_1, .., C_n\}$*
- *for every $j > n+1$, C_j is the resolvent of C_{j-1} and another $C \in \{C_1, .., C_n\}$*
- *every resolution step takes the form*

$$\begin{array}{c} L' \vee C' \\ \neg L'' \vee C'' \end{array} \quad \rightsquigarrow \quad (C' \vee C'')(MGU(L', L''))$$

Properties: SLD resolution is

- linear
- input
- directed
- ordered

LUSH resolution

The selection rule

In SLD, the rule requires the factor to be the first literal in both clauses

- as a consequence, the goal clause does not contain a non-negated literal and has to resolve with a clause whose first literal in non-negated

LUSH: Linear resolution with Unrestricted Selection for Horn clauses

- linear, input and directed but not ordered: every literal can be resolved with any other

LUSH resolution

Example: goal $C_7 : \neg C(x) \vee \neg E(x)$

$C_1 : D(x) \vee \neg A(x)$ $\quad$ $C_2 : E(x) \vee \neg A(x)$

$C_3 : A(a)$ $\quad$ $C_4 : B(a)$

$C_5 : C(x) \vee \neg D(x) \vee \neg B(x)$ $\quad$ $C_6 : B(x) \vee \neg D(x) \vee \neg C(x)$

SLD

- C_7, with C_5
- $\neg D(x) \vee \neg B(x) \vee \neg E(x)$, with C_1
- $\neg A(x) \vee \neg B(x) \vee \neg E(x)$, with C_3
- $\neg B(a) \vee \neg E(a)$, with C_4
- $\neg E(a)$, with C_2
- $\neg A(a)$, with C_3
- $\square$

LUSH

- C_7, with C_2
- $\neg A(x) \vee \neg C(x)$, with C_3
- $\neg C(a)$, with C_5
- $\neg D(a) \vee \neg B(a)$, with C_1
- $\neg A(a) \vee \neg B(a)$, with C_3
- $\neg B(a)$, with C_4
- $\square$

SLD resolution

Lemma

The support set of a set of Horn Clauses is satisfiable

Proof.

1. the clauses of the support set have a non-negated literal
2. an interpretation which assigns **t** to such literals makes the set true

Corollary

If there exists a refutation of a set of Horn clauses, then there exists a directed refutation on the support set

Lemma

If there exists a LUSH refutation of a set of Horn clauses, then there exists an SLD refutation of the same set

SLD resolution

Theorem

SLD resolution is complete for Horn clauses: if a set of Horn clauses is unsatisfiable, then there exists an SLD refutation for it

Proof.

1. $UNSAT(H)$
1. there exists a refutation of H (completeness of resolution)
2. there exists a directed refutation $\mathcal{R}$ (the support set is satisfiable)
   - every step involves a goal clause or an intermediate resolvent
3. $\mathcal{R}$ is an input refutation
   - every step requires a clause with a non-negated literal, i.e., a support clause
   - support clauses are input clauses
4. if there exists an input refutation, then there exists a linear input one $\mathcal{R}'$
   - $\mathcal{R}'$ is directed, input and linear, that is, LUSH
5. there exists an SLD refutation $\mathcal{R}''$ (lemma above)

SLD resolution

When studying a set of Horn clauses

- possible refutations can be restricted to SLD refutations
- search trees can be restricted to SLD search trees for $\Box$

Depth and breadth

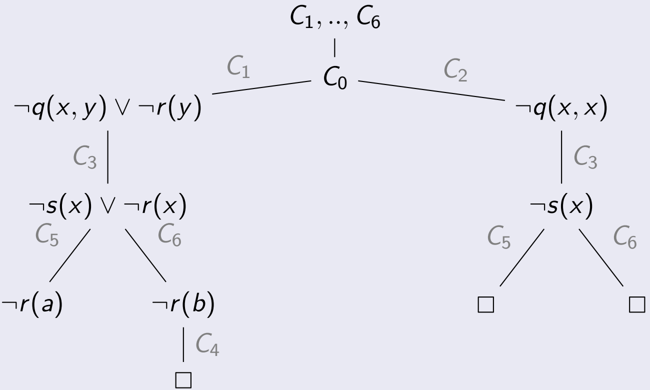
- breadth-first SLD is complete, depth-first is not
- in the depth-first approach, it is crucial how to choose the order for selecting support clauses to be resolved with the current goal clause
  - *computation function*
- depending on the search strategy
  - some refutations are not found
  - some derivations do not terminate

SLD resolution

Example

$C_1 : p(y) \vee \neg q(x,y) \vee \neg r(y)$ $\quad C_2 : p(x) \vee \neg q(x,x)$ $\quad C_3 : q(x,x) \vee \neg s(x)$

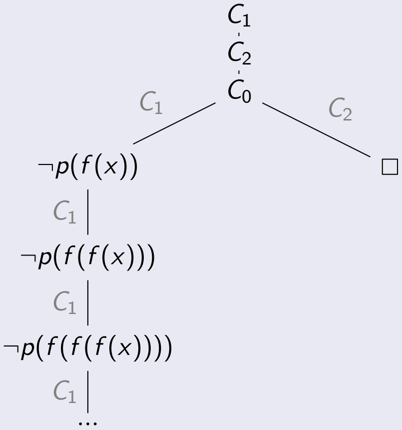
$C_4 : r(b)$ $\quad C_5 : s(a)$ $\quad C_6 : s(b)$ $\quad C_0 : \neg p(x)$



SLD resolution

Example: $C_1 : p(x) \vee \neg p(f(x))$, $C_2 : p(a)$, $C_0 : \neg p(y)$

- a depth search with a computation function which chooses the first support clause does not terminate



SLD resolution

Example: $C_1 : p(x) \vee \neg p(f(x))$, $C_2 : p(a)$, $C_0 : \neg p(y)$

- but a refutation can be obtained by changing the order of the support clauses (C_2 before C_1)