

CODE OBFUSCATION

DEFENSE STRATEGIES

Roberto Giacobazzi

Dipartimento di Informatica
Università degli Studi di Verona
Italy

ASP 2009

THE SOURCE

Most of the slides are taken from: Ch. 4, 5 and 6 of

Surreptitious Software: Obfuscation, Watermarking, and Tamperproofing for Software Protection

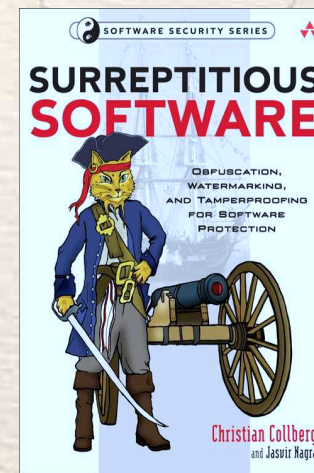
Christian Collberg

Jasvir Nagra

ISBN-10: 0321549252

ISBN-13: 9780321549259

Addison-Wesley Professional 2010, 792 pp.



and

Roberto Giacobazzi. **Hiding Information in Completeness Holes.**

The 6th IEEE International Conferences on
Software Engineering and Formal Methods, SEFM'08, pages 7-20, IEEE Press.

DEFENSE STRATEGIES

Actions in **Frames**: ... let us see the *Bofo marinus*



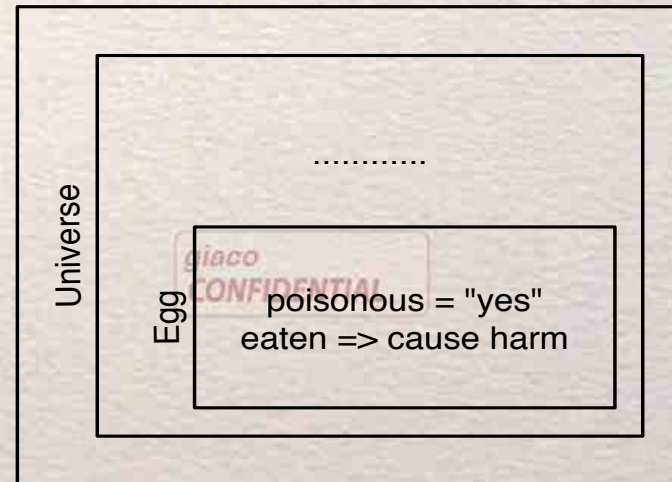
One or more slots: *name = value*



Slots may contain other slots



Conditional actions \Rightarrow can take place



DEFENSE STRATEGIES: THE PRIMITIVES

We define 10 primitives which can be **composed** to design a generic defense strategy:



Cover

giaco
CONFIDENTIAL

Duplicate



Split and Merge



Reorder



Map

giaco
CONFIDENTIAL

Indirect

giaco
CONFIDENTIAL

Mimic

giaco
CONFIDENTIAL

Advertise



Detect/Respond

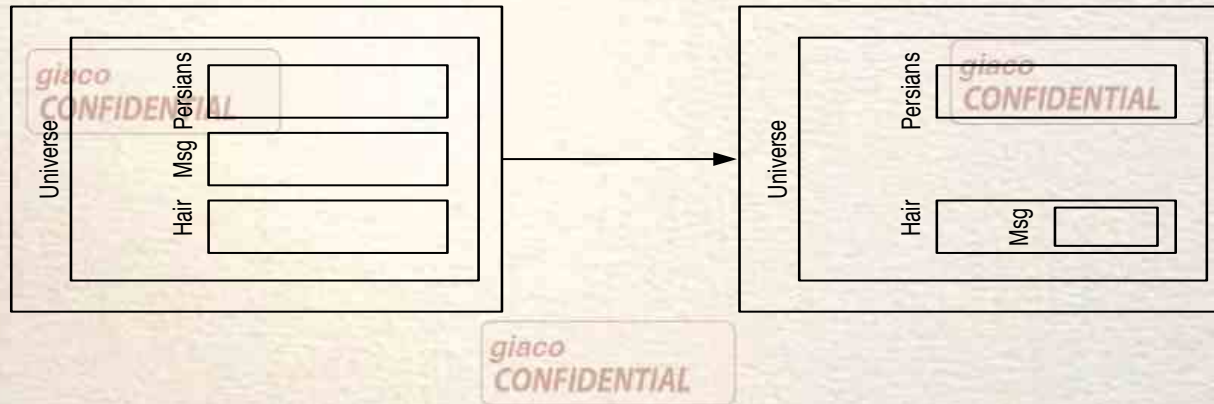


Dynamic

giaco
CONFIDENTIAL

DEFENSE STRATEGIES: *Cover*

Given two frames x and y make x an element of y :



↪ Cover can be applied multiple times to hide information in an inner level

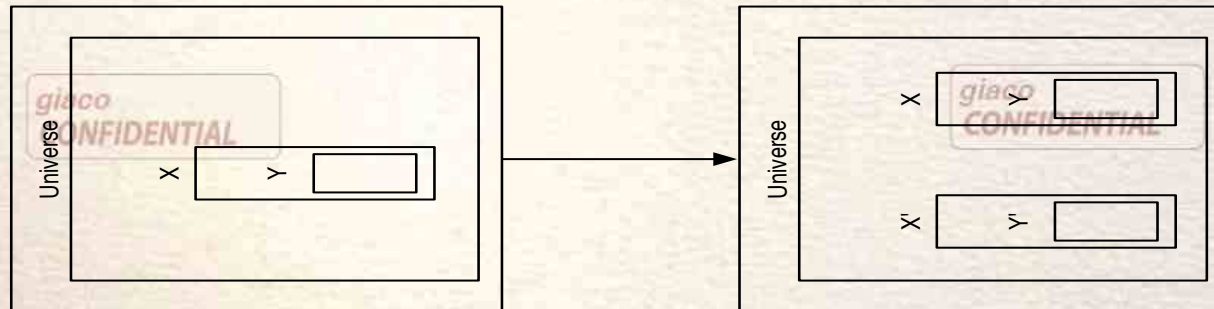
↪ Hiding = covering = obscuring

↪ Typical examples:

- ✓ hiding keys or SW in hardened boxes (Military)
- ✓ hiding watermarks in standard data-structures
- ✓ hiding watermarks in images of media

DEFENSE STRATEGIES: *Duplicate*

Given a frame x , create a deep copy of x (keeping names unique):



➡ **Idea 1:** Copy as *decoy*: make the universe larger and harder to scan

➡ **Idea 2:** Copy as *reduplication*: make the universe larger, full of your copies (signatures)

➡ Typical examples:

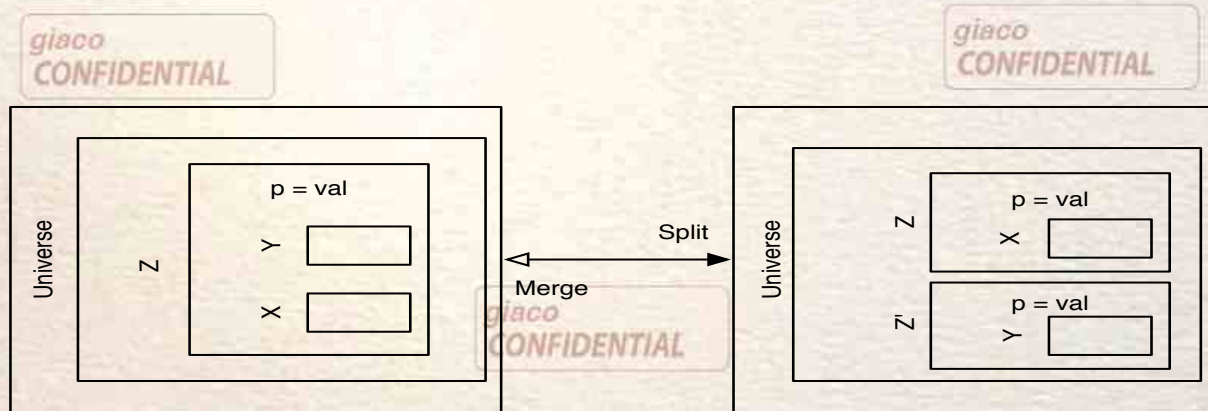
- ✓ reduplication for protecting its own DNA
- ✓ dummy targets for confusing adversaries

```
int THE_WATERMARK_IS_HERE = 666
```

- ✓ obfuscate f by $f' := Duplicate(f)$, $f'' := Obf(f')$ and call f and f''

DEFENSE STRATEGIES: *Split & Merge*

Given a predicate π and a frame z create a new frame z' such that z' has all the properties of z and $\forall x \in z'. \pi(x)$. Merge is set union of frames (and related properties):



Typically used in combination: take two functions f and g :
 $Split(f) = (f_1, f_2)$ and $Split(g) = (g_1, g_2)$, then:

$$fg = Merge(f_1, g_2) =$$

$$\lambda x, y. \text{ if } x = 1 \text{ then } f_1(y) \text{ else } g_2(y)$$

$$gf = Merge(f_2, g_1) =$$

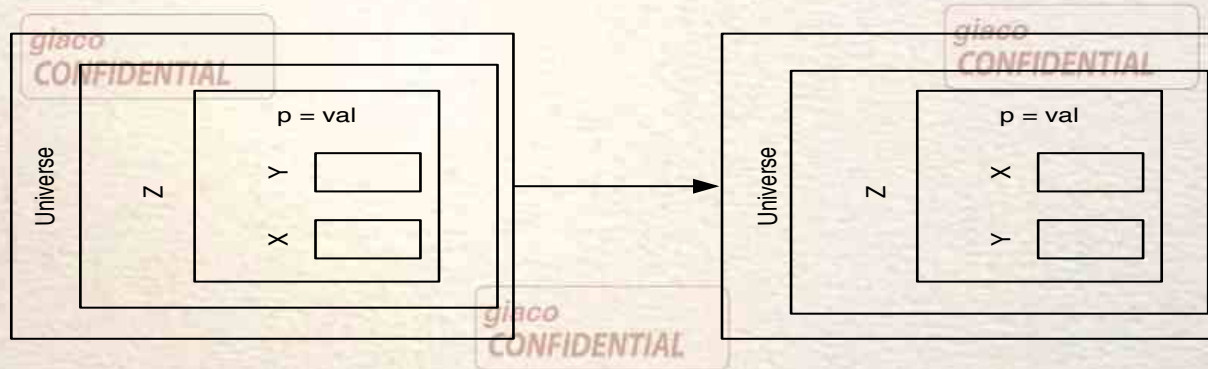
$$\lambda x, y. \text{ if } x = 2 \text{ then } f_2(y) \text{ else } g_1(y)$$

\Rightarrow call $f(y) \rightarrow$

$$\left[\begin{array}{l} x = 1 \\ fg(x, y) \\ x ++ \\ gf(x, y) \end{array} \right.$$

DEFENSE STRATEGIES: *Reorder*

Given a frame z and a permutation function f , reorder the elements of z according to f :

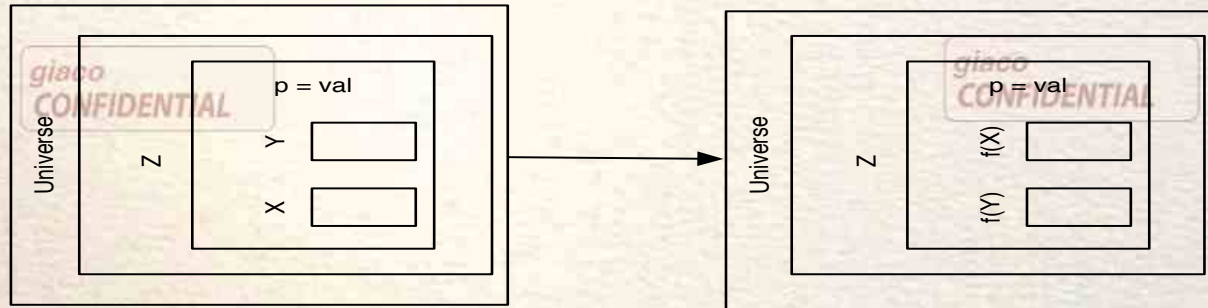


➡ Used in early SW watermarking e.g., by reordering basic blocks in CFG

➡ Used in code obfuscation and metamorphism by reordering basic blocks in CFG

DEFENSE STRATEGIES: *Map*

Given a frame x and a function f , replace every element e in x with $f(e)$:



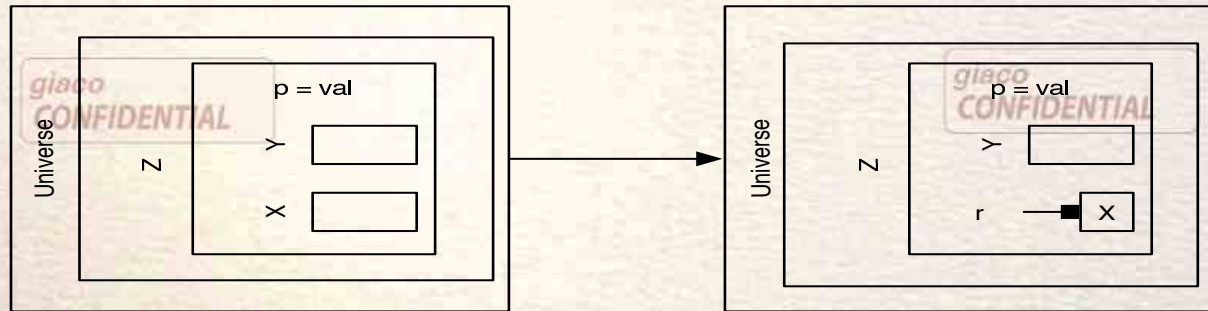
Implements **Security-through-obscurity**: translation, crypto, etc.



Protect confidentiality by name obfuscation (translation): variables, functions, data

DEFENSE STRATEGIES: *Indirect*

Given a frame x add an indirect reference r to x :



The cost of following pointers makes the analysis harder!!

```
void fun1(){}
void fun2(){
    fun1();
}
```

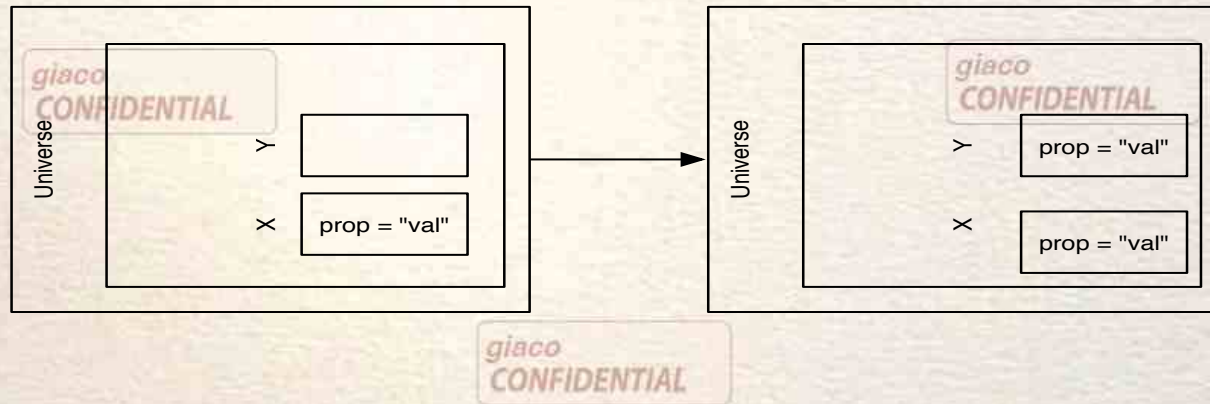
```
void fun1(){}
void (*ref1)()=&fun1;
void (*ref2)()=&ref1;
void fun2(){
    (**ref2)();
}
```

The diagram shows the code from the previous block with arrows indicating pointer resolution. A red dot on the line `(**ref2)();` has an arrow pointing to a red dot on the line `(*ref2)()=&ref1;`. This second dot has an arrow pointing to a red dot on the line `(*ref1)()=&fun1;`. This third dot has an arrow pointing to a red dot on the line `fun1(){}.`

Combined with *map* you can hide references.

DEFENSE STRATEGIES: *Mimic*

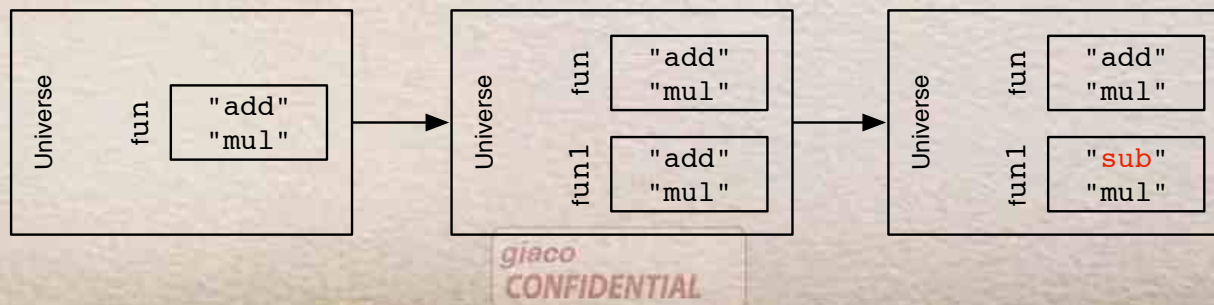
Given two frames x and y , where x holds a property prop , copy prop into y :



Common in wild-life (animals)

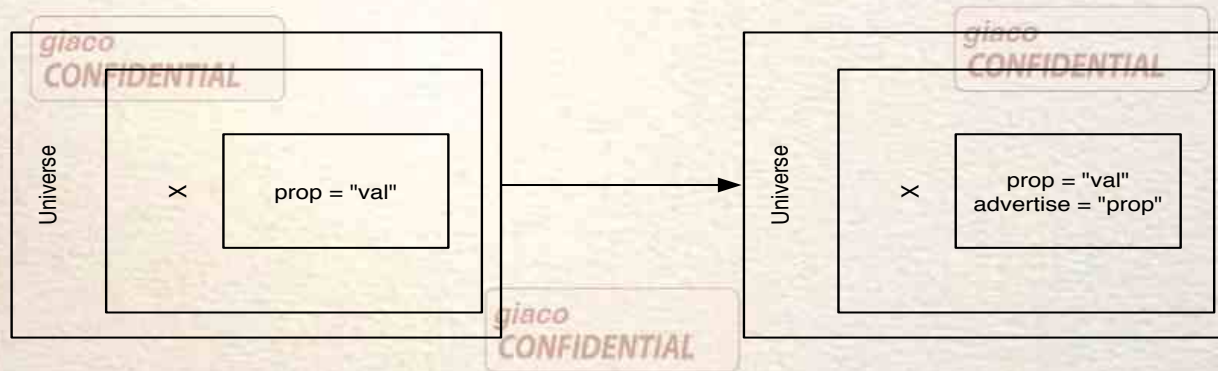


Fundamental for **stealthy**: the new (watermarked) code must resemble the same a standard code. Here is a static watermarking:



DEFENSE STRATEGIES: *Advertise*

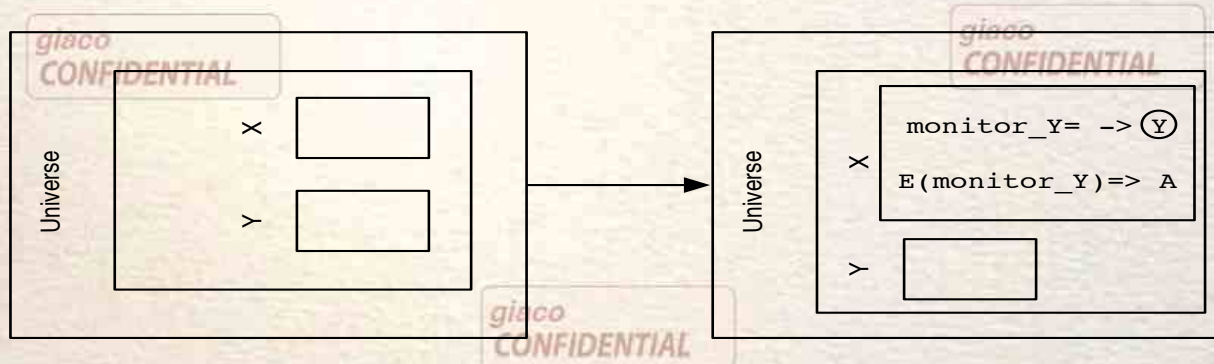
Given a frame x with a property `prop` add a property `advertise` to x with value `prop`:



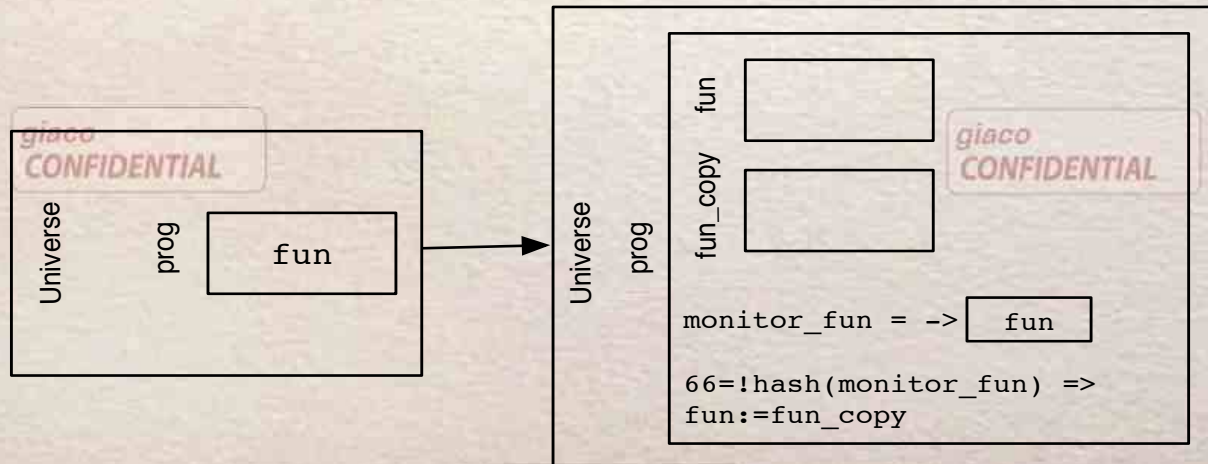
- ➡ Opposite to **security-through-obscurity**
- ➡ Openly display a situation in order to discourage attacks
- ➡ Example by false advertising: **Say that P is watermarked when it is not!**

DEFENSE STRATEGIES: *Detect/Respond*

Given two frames x and y add a demon to x that executes action A if event E happens to y :



Typical in tamper-proofing: *Duplictate* \rightarrow *Map* \rightarrow *Detect/Respond*



DEFENSE STRATEGIES: *Dynamic*

Iterate a primitive f over a frame x generating a sequence (finite or infinite) of frames:

$$f(x) \longrightarrow f(f(x)) \longrightarrow f(f(f(x))) \longrightarrow \dots$$

where

$f \in$
{*Cover, Duplicate, Split, Merge, Reoder, Map, Indirect, Mimic, Advertise, Detect/Respond*}



Dynamic change for confusing the attacker: *Polymorphism* and *Metamorphism*



Useful in polymorphic malware: **dynamically decrypt (map) chunks of code, execute, reencrypt (map)**

THE PROBLEM: OBFUSCATION VS DIVERSITY

F. Cohen, Operating systems protection through program evolution, 1993



Generate syntactically different semantic equivalent programs



P and Q are syntactically different if $P \neq Q$



P and Q are semantic equivalent if $P \equiv Q$ i.e.,

- ✓ If $\llbracket P \rrbracket(x) \downarrow$ and $\llbracket Q \rrbracket(x) \downarrow$ then $\llbracket P \rrbracket(x) = \llbracket Q \rrbracket(x)$
- ✓ If $\llbracket P \rrbracket(x) \uparrow$ or $\llbracket Q \rrbracket(x) \uparrow$ then $\llbracket P \rrbracket(x) \uparrow$ and $\llbracket Q \rrbracket(x) \uparrow$



Given a program P :

$\mathfrak{D}(P) \stackrel{\text{def}}{=} \left\{ Q \mid P \equiv Q \wedge P \neq Q \right\}$ is a non recursive set!

LAYOUT OBFUSCATION

Standard and easy methods for making your code diverse:



Change your code by substituting equivalent expressions:

```
giaco CONFIDENTIAL
```

$$y = x * 42 \implies \begin{aligned} &y = x \ll 5; \\ &y += x \ll 3; \\ &y += x \ll 1; \end{aligned}$$

```
giaco CONFIDENTIAL
```



Reordering code: break **locality** which is a typical principle used in reverse engineering!



Identifier renaming: ... that is really too easy!!!

OBFUSCATION BY INTERPRETATION

see Y. Futamura, *Partial Evaluation of Computation Process*, 1971

Consider two programming languages (abstract machines): \mathcal{S} and \mathcal{T} with common data



Interpreter: $\forall \ell \in \{\mathcal{S}, \mathcal{T}\}$

- ✓ $\text{int}^{\mathcal{S}} : \mathcal{S}.prog \times data \longrightarrow data \cup \{\uparrow\}, \text{int}^{\mathcal{S}} \in \mathcal{S}.prog$
- ✓ $\text{int}^{\mathcal{T}} : \mathcal{T}.prog \times data \longrightarrow data \cup \{\uparrow\}, \text{int}^{\mathcal{T}} \in \mathcal{S}.prog$
- ✓ $\forall P \in \ell.prog, \forall d \in data : \llbracket P \rrbracket^{\ell}(d) = \llbracket \text{int}^{\ell} \rrbracket^{\mathcal{S}}(p, d)$



Code specializer: $\forall \ell \in \{\mathcal{S}, \mathcal{T}\}$

- ✓ $\text{spec}^{\mathcal{S}} : \mathcal{S}.prog \times data \longrightarrow \mathcal{S}.prog, \text{spec}^{\mathcal{S}} \in \mathcal{S}.prog$
- ✓ $\text{spec}^{\mathcal{T}} : \mathcal{S}.prog \times data \longrightarrow \mathcal{T}.prog, \text{spec}^{\mathcal{T}} \in \mathcal{S}.prog$
- ✓ $\forall P \in \mathcal{S}.prog, \forall d, s \in data : \llbracket P \rrbracket^{\mathcal{S}}(s, d) = \llbracket \llbracket \text{spec}^{\ell} \rrbracket^{\mathcal{S}}(P, s) \rrbracket^{\ell}(d)$



Idea:

\mathcal{S} is the source language (open)

\mathcal{T} is the secret architecture (hidden)

OBFUSCATION BY INTERPRETATION

see Y. Futamura, *Partial Evaluation of Computation Process*, 1971

Obfuscated code for P is a compiled code from \mathcal{S} to \mathcal{T} and back to \mathcal{S} :

$$\begin{aligned}
 \llbracket P \rrbracket^{\mathcal{S}}(d) &= \llbracket \text{int}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(P, d) \\
 &= \llbracket \llbracket \text{spec}^{\mathcal{T}} \rrbracket^{\mathcal{S}}(\text{int}^{\mathcal{S}}, P) \rrbracket^{\mathcal{T}}(d) \\
 &= \llbracket \text{int}^{\mathcal{T}} \rrbracket^{\mathcal{S}}(\llbracket \text{spec}^{\mathcal{T}} \rrbracket^{\mathcal{S}}(\text{int}^{\mathcal{S}}, P), d) \\
 &= \llbracket \llbracket \text{spec}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(\text{int}^{\mathcal{T}}, \llbracket \text{spec}^{\mathcal{T}} \rrbracket^{\mathcal{S}}(\text{int}^{\mathcal{S}}, P)) \rrbracket^{\mathcal{S}}(d)
 \end{aligned}$$

$$\text{obf}(P) = \llbracket \text{spec}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(\text{int}^{\mathcal{T}}, \llbracket \text{spec}^{\mathcal{T}} \rrbracket^{\mathcal{S}}(\text{int}^{\mathcal{S}}, P)) \in \mathcal{S}.prog$$



In order to attack $\text{obf}(P)$ you need to understand the relation between \mathcal{S} and \mathcal{T}



$\text{obf}(P)$ may run 10-100 time slower!!

OBFUSCATOR BY INTERPRETATION

see Y. Futamura, *Partial Evaluation of Computation Process*, 1971

An obfuscator can be generated by further specializing the specialized by an interpreter:

$$\begin{aligned}
 \text{obf}(P) &= \llbracket \text{spec}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(\text{int}^{\mathcal{T}}, \llbracket \text{spec}^{\mathcal{T}} \rrbracket^{\mathcal{S}}(\text{int}^{\mathcal{S}}, P)) \\
 &= \llbracket \llbracket \text{spec}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(\text{spec}^{\mathcal{S}}, \text{int}^{\mathcal{T}}) \rrbracket^{\mathcal{S}}(\llbracket \text{spec}^{\mathcal{T}} \rrbracket^{\mathcal{S}}(\text{int}^{\mathcal{S}}, P)) \\
 &= \llbracket \llbracket \text{spec}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(\text{spec}^{\mathcal{S}}, \text{int}^{\mathcal{T}}) \rrbracket^{\mathcal{S}}(\llbracket \llbracket \text{spec}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(\text{spec}^{\mathcal{T}}, \text{int}^{\mathcal{S}}) \rrbracket^{\mathcal{S}}(P))
 \end{aligned}$$

$$\text{obf} = \llbracket \text{spec}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(\text{spec}^{\mathcal{T}}, \text{int}^{\mathcal{S}}); \llbracket \text{spec}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(\text{spec}^{\mathcal{S}}, \text{int}^{\mathcal{T}}) \in \mathcal{S}.prog$$



By modifying \mathcal{T} you can generate different obfuscators by interpretation!



Highly modular but extremely expensive (x100 slowdown by emulation)

OBFUSCATOR GENERATOR BY INTERPRETATION

see Y. Futamura, *Partial Evaluation of Computation Process*, 1971

A generator of obfuscators can be obtained by further specializing the specializer:

$$\begin{aligned}
 \text{obf}(P) &= \llbracket \text{spec}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(\text{int}^{\mathcal{T}}, \llbracket \text{spec}^{\mathcal{T}} \rrbracket^{\mathcal{S}}(\text{int}^{\mathcal{S}}, P)) \\
 &= \llbracket \llbracket \text{spec}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(\text{spec}^{\mathcal{S}}, \text{int}^{\mathcal{T}}) \rrbracket^{\mathcal{S}}(\llbracket \text{spec}^{\mathcal{T}} \rrbracket^{\mathcal{S}}(\text{int}^{\mathcal{S}}, P)) \\
 &= \llbracket \llbracket \text{spec}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(\text{spec}^{\mathcal{S}}, \text{int}^{\mathcal{T}}) \rrbracket^{\mathcal{S}}(\llbracket \llbracket \text{spec}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(\text{spec}^{\mathcal{T}}, \text{int}^{\mathcal{S}}) \rrbracket^{\mathcal{S}}(P))
 \end{aligned}$$

$$\text{obf} = \llbracket \text{spec}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(\text{spec}^{\mathcal{T}}, \text{int}^{\mathcal{S}}); \llbracket \text{spec}^{\mathcal{S}} \rrbracket^{\mathcal{S}}(\text{spec}^{\mathcal{S}}, \text{int}^{\mathcal{T}}) \in \mathcal{S}.prog$$



By modifying \mathcal{T} you can generate different obfuscators by interpretation

COMBINING OBFUSCATORS: DIVERSIFICATION

Assume you have a set of obfuscation strategies: $\{\mathcal{T}_1, \dots, \mathcal{T}_n\}$



Let $\text{prob}[i]$ be the probability of applying \mathcal{T}_i



Let *thresholdspace* be the bound to the space of obfuscated code

```
input P;
while space(P) ≤ thresholdspace {
  x ← y ← 0
  a ← random in [0, 1]
  while x ≤ a {
    y ← y + 1;
    x ← x + prob[y]
  };
  apply( $\mathcal{T}_y$ , P)
}
```

COMBINING OBFUSCATORS: DIVERSIFICATION II

Assume you have a set of obfuscation strategies: $\mathcal{T} = \{\mathcal{T}_1, \dots, \mathcal{T}_n\}$



P consists of objects $\langle s_1, \dots \rangle$ (routines, modules, etc)



Let $\text{prio}[s_i]$ is the importance of protecting object s_i



acceptCost is the maximum execution penalty allowed



reqObf is the amount of obfuscation required

```
input  $P = \langle s_1, \dots \rangle$ ;  
while !done(acceptCost, reqObf) {  
     $s \leftarrow \text{max in prio}$ ;  
     $t \leftarrow \text{selectTrans}(s, \mathcal{T})$ ;  
    apply( $t, s$ );  
    update(prio[ $s$ ])  
};
```

COMBINING OBFUSCATORS: DEPENDENCIES

Assume you have a set of obfuscation strategies: $\mathcal{T} = \{\mathcal{T}_1, \dots, \mathcal{T}_n\}$



The order by which obfuscations are applied may be relevant!



Idea: build a FSA whose language *describe* the acceptable order of obfuscations:

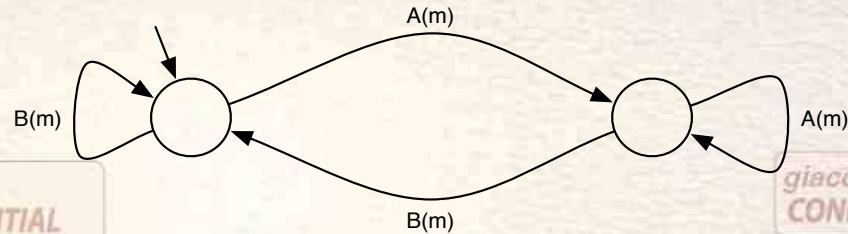
$$P = m() \text{ and } \begin{cases} \text{Obf.level} & 0.6 \\ \text{Perf.critical} & 0.2 \end{cases} \quad \mathcal{T} = \{\mathcal{T}_1, \mathcal{T}_2\}$$

where

after \mathcal{T}_1 do \mathcal{T}_2

	Transf	Potency	Degradation
\mathcal{T}_1	1.0	0.9	
\mathcal{T}_2	0.5	0.3	

COMBINING OBFUSCATORS: DEPENDENCIES



The accepted language of the FSA \mathcal{G} is: $(B(m) + A(m)^+ B(m))^*$



The best sequences can be chosen by using weights on edge:

$$\forall t \in \mathcal{T}, \forall s \in \mathcal{P} :$$

$$\text{weigh}(t, s) = \mathcal{F}(\text{Obf.level}(s), \text{Perf.critical}(s), \text{Potency}(t), \text{Degradation}(t))$$

where $\mathcal{F} : [0, 1]^4 \longrightarrow [0, 1]$ combines the single weights!



Goal: maximize the weight along some path!

DEFINITIONS



Let us have some secret σ in P that we want to hide: $\mathcal{D}(P)$. What is σ ?

- ✓ The source code of P ;
- ✓ The module organization of P ;
- ✓ Function names
- ✓ Location of critical functions (e.g., license check)
- ✓ The value of particular data (crypto keys, license etc)

giaco
CONFIDENTIAL

Let $\mathcal{D} : \mathbb{P} \rightarrow \mathbb{P}$ be a program transformation. $\mathcal{D}(P)$ is an obfuscation of P if:

- ✓ $P \equiv_{\alpha} \mathcal{D}(P)$;
- ✓ σ is *hidden* in $\mathcal{D}(P)$.

giaco
CONFIDENTIALgiaco
CONFIDENTIAL

To understand how *hidden* is σ we need to know how to *attack* σ

DEFINITIONS



Let $\mathcal{O} : \mathbb{P} \longrightarrow \mathbb{P}$ be an obfuscation. $\mathcal{O}(P)$ is the obfuscation of $P \in \mathbb{P}$



Let \mathcal{A} be an analysis for the language \mathbb{P} :

- ✓ If $\sigma = \mathcal{A}(P)$ then \mathcal{A} **reveals** σ ;
- ✓ $P \equiv_{\alpha} \mathcal{O}(P)$;

What is the relation between \mathcal{A} and \mathcal{O} ?

- ✓ \mathcal{O} is *effective* if $\mathcal{A}(\mathcal{O}(P)) \neq \sigma$ or $\mathcal{A}(\mathcal{O}(P))$ is harder than $\mathcal{A}(P)$;
- ✓ \mathcal{O} is *ineffective* if $\mathcal{A}(\mathcal{O}(P)) = \sigma$;
- ✓ \mathcal{O} is *defective* if $\mathcal{A}(\mathcal{O}(P)) \neq \sigma$ and $\mathcal{A}(\mathcal{O}(P))$ is easier than $\mathcal{A}(P)$.

A good obfuscator either hides σ with respect to \mathcal{A} or makes \mathcal{A} harder!

DEFINITIONS: POTENCY



Let $\mathcal{D} : \mathbb{P} \longrightarrow \mathbb{P}$ be an obfuscation. $\mathcal{D}(P)$ is the obfuscation of $P \in \mathbb{P}$



Let $\{\mathcal{A}_1, \dots, \mathcal{A}_n\}$ be a set of program analysis for the language \mathbb{P} :

\mathcal{D} is *potent* for P if

- ✓ $\exists \mathcal{A}_i \in \{\mathcal{A}_1, \dots, \mathcal{A}_n\}$ such that \mathcal{D} is effective for P and \mathcal{A}_i ;
- ✓ $\forall \mathcal{A}_j \in \{\mathcal{A}_1, \dots, \mathcal{A}_n\} \wedge i \neq j$ \mathcal{D} is not defective for P .

Effectiveness deals with: precision and complexity!!

MEASURES OF POTENCY



Typical metric measures

- ✓ **Program length** $\mathcal{D}(P)$ increases the operators and operands in P
- ✓ **Cyclomatic complexity** $\mathcal{D}(P)$ increases the number of predicates in P
- ✓ **Nesting complexity** $\mathcal{D}(P)$ increases the nesting level of conditionals in P
- ✓ **Data-flow complexity** $\mathcal{D}(P)$ increases the inter-block reference
- ✓ **Fan-in/Fan-out complexity** $\mathcal{D}(P)$ increases the number of formal parameter and data structured referenced by P
- ✓ **Data-structure complexity** $\mathcal{D}(P)$ increases the complexity of static data structures (arrays and records) declared in P
- ✓ **OO metric** $\mathcal{D}(P)$ increases the number of methods, the depth in inheritance tree, the number of sub classes, and methods that can be generated in response to a message sent to an object

Is there a more general notion of potency?

ABSTRACT INTERPRETATION

[Cousot & Cousot '79]



A program P and a domain of computation for $P: C$



Semantic specification (interpreter): $\llbracket P \rrbracket : C \longrightarrow C$



(Approximate) observable properties: $\rho \in uco(C)$



DERIVE A SOUND APPROXIMATE SPECIFICATION $\llbracket P \rrbracket^\#$

$$\rho(\llbracket P \rrbracket(x)) \leq \llbracket P \rrbracket^\#(x)$$



THE LIMIT CASE: COMPLETENESS

$$\rho(\llbracket P \rrbracket(x)) = \llbracket P \rrbracket^\#(x) \text{ iff } \rho(\llbracket P \rrbracket(x)) = \rho(\llbracket P \rrbracket(\rho(x)))$$

COMPLETENESS IN ABSTRACT INTERPRETATION

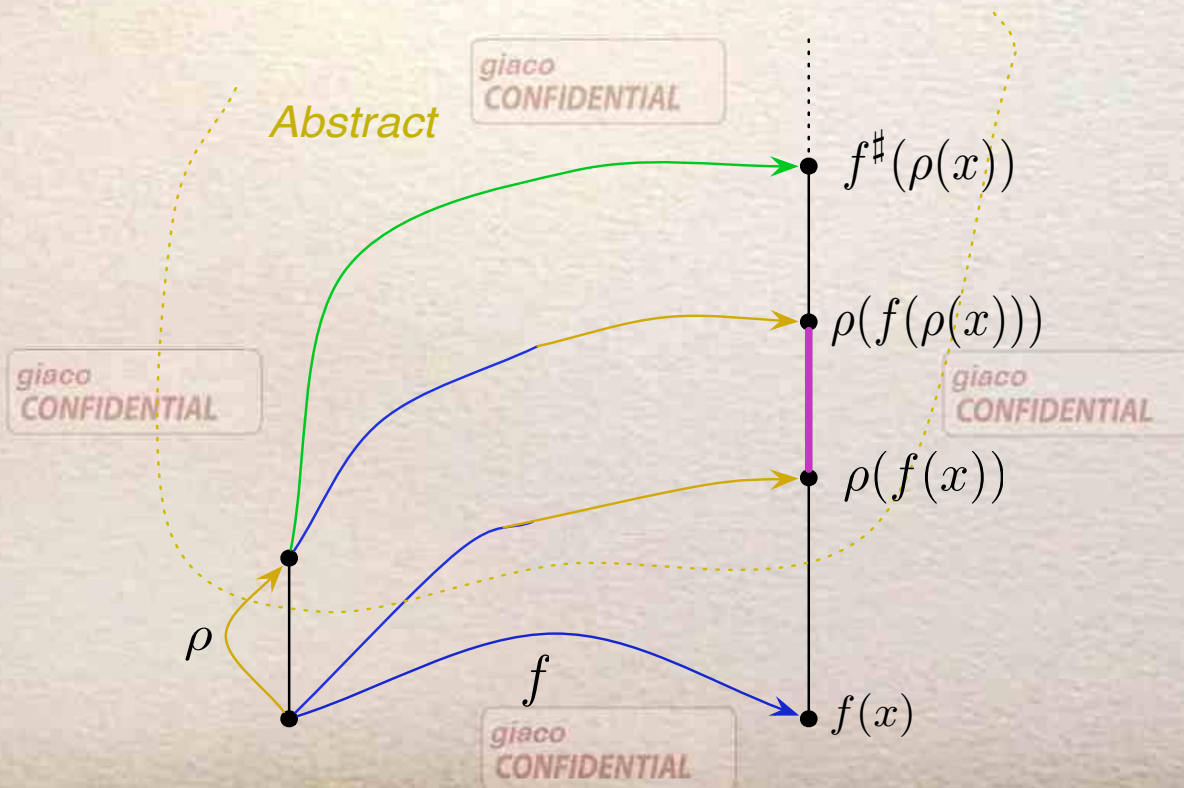


BACKWARD SOUNDNESS:

NO INFORMATION IS LOST BY APPROXIMATING THE INPUT/OUTPUT



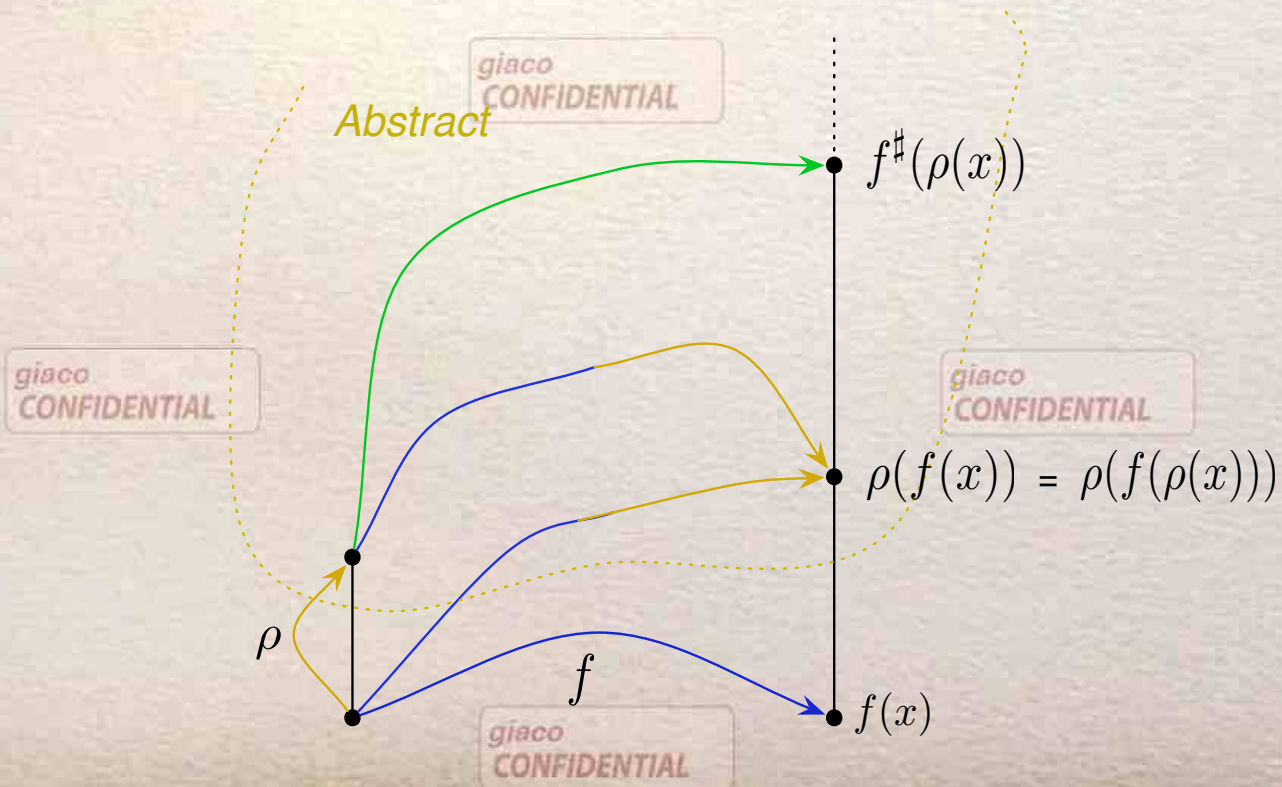
$$\rho \circ f \leq \rho \circ f \circ \rho$$

giaco
CONFIDENTIALgiaco
CONFIDENTIAL

COMPLETENESS IN ABSTRACT INTERPRETATION

⇨ BACKWARD COMPLETENESS:
NO LOSS OF PRECISION IS ACCUMULATED BY APPROXIMATING THE INPUT

⇨ $\rho \circ f = \rho \circ f \circ \rho$

giaco
CONFIDENTIALgiaco
CONFIDENTIAL

COMPLETENESS IN ABSTRACT INTERPRETATION

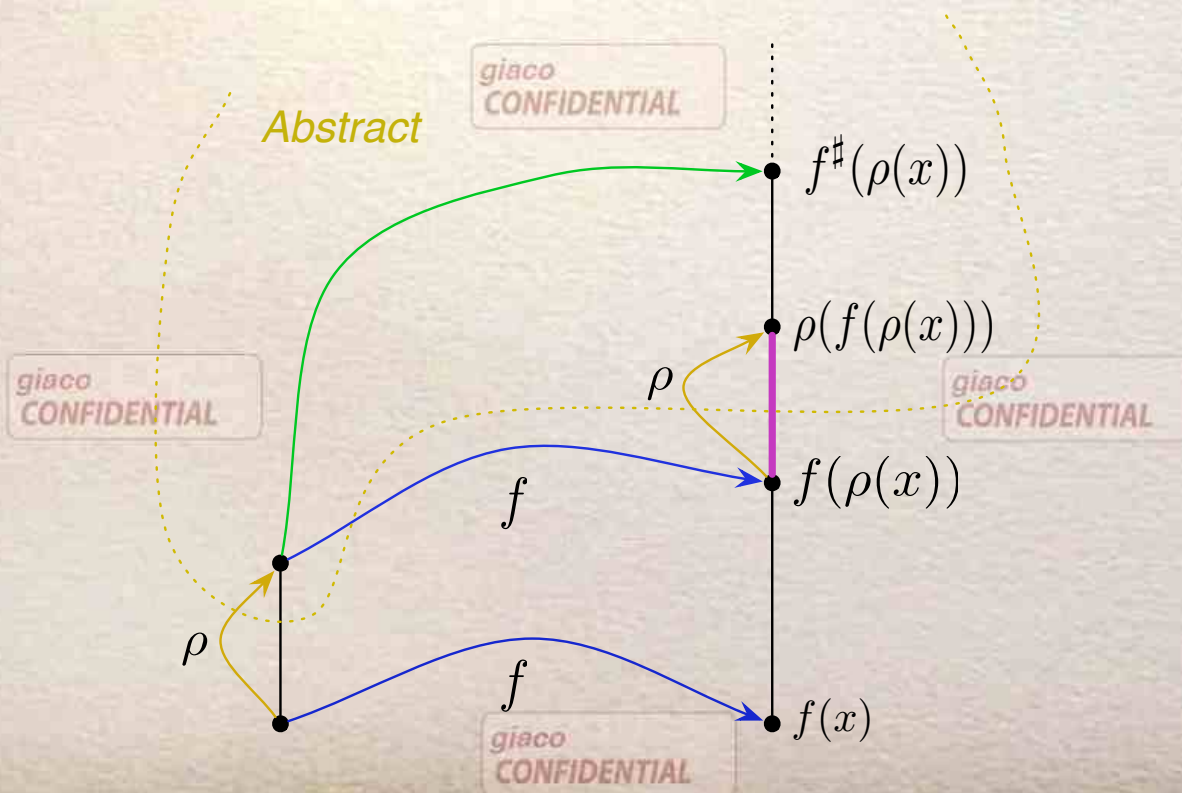


FORWARD COMPLETENESS:

NO INFORMATION IS LOST BY APPROXIMATING THE OUTPUT



$$f \circ \rho \leq \rho \circ f \circ \rho$$

giaco
CONFIDENTIALgiaco
CONFIDENTIAL

COMPLETENESS IN ABSTRACT INTERPRETATION

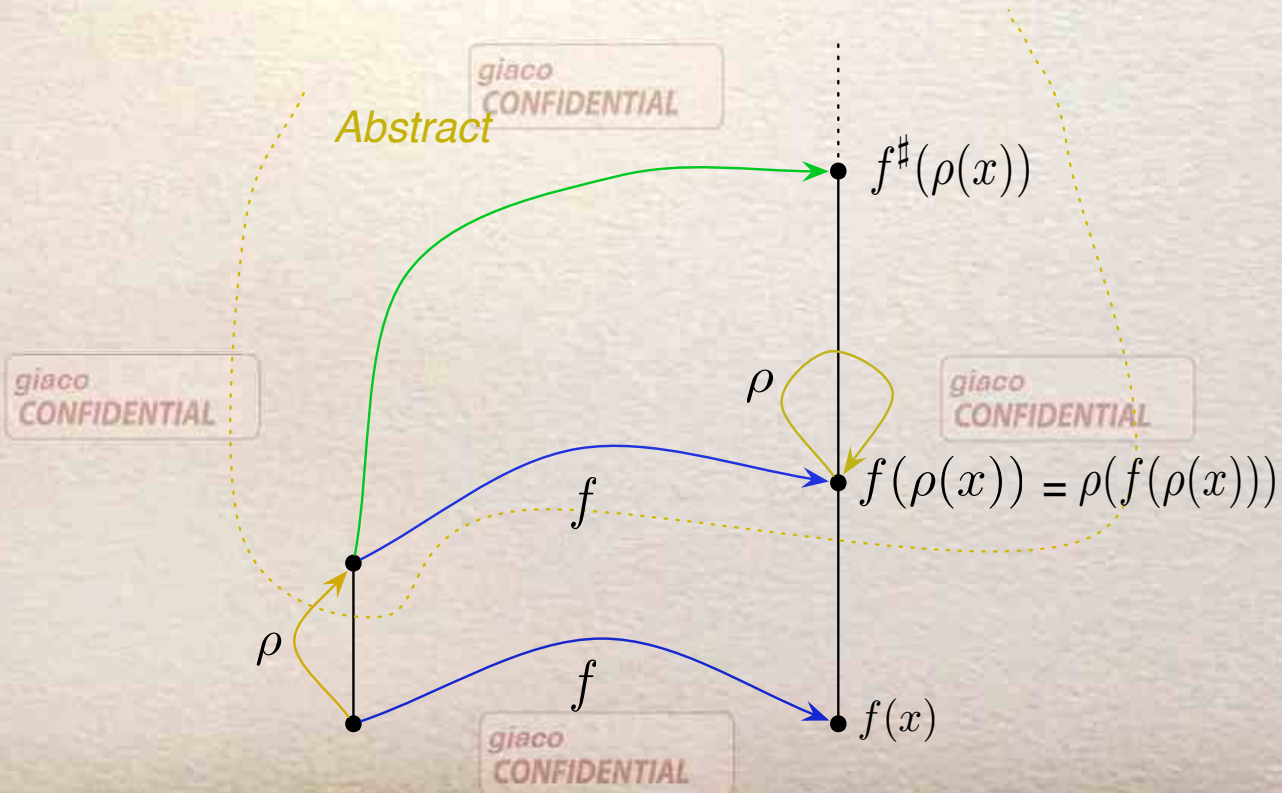


FORWARD COMPLETENESS:

NO INFORMATION IS LOST BY APPROXIMATING THE OUTPUT



$$f \circ \rho = \rho \circ f \circ \rho$$

giaco
CONFIDENTIALgiaco
CONFIDENTIAL

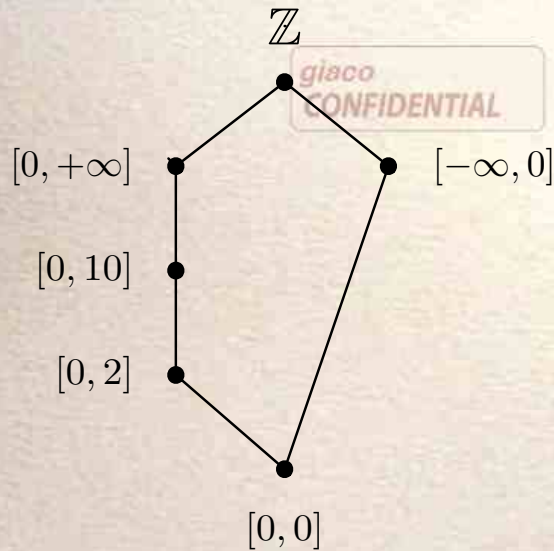
giaco
CONFIDENTIAL

giaco
CONFIDENTIAL

giaco
CONFIDENTIAL

A CLASSICAL EXAMPLE

A SIMPLE EXAMPLE IN INTERVAL ANALYSIS



A simple domain of intervals

giaco
CONFIDENTIAL

giaco
CONFIDENTIAL

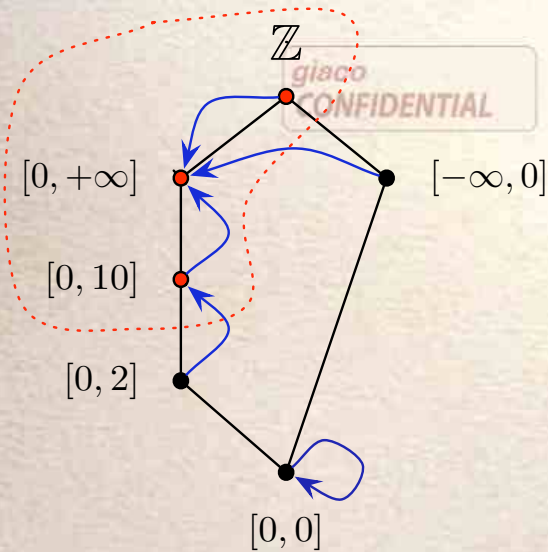
giaco
CONFIDENTIAL

giaco
CONFIDENTIAL

giaco
CONFIDENTIAL

A CLASSICAL EXAMPLE

A SIMPLE EXAMPLE IN INTERVAL ANALYSIS



A simple domain of intervals



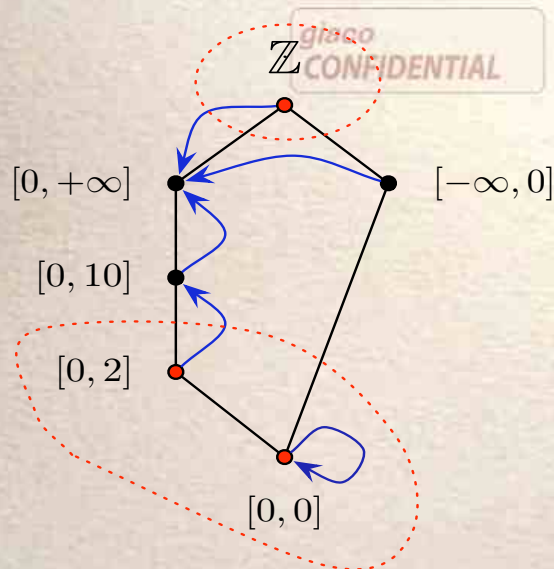
$$sq(X) = \left\{ x^2 \mid x \in X \right\}$$



$\{\mathbb{Z}, [0, +\infty], [0, 10]\}$ is Forward but not Backward complete

A CLASSICAL EXAMPLE

A SIMPLE EXAMPLE IN INTERVAL ANALYSIS



A simple domain of intervals



$$sq(X) = \left\{ x^2 \mid x \in X \right\}$$



$\{\mathbb{Z}, [0, +\infty], [0, 10]\}$ is Forward but not Backward complete



$\{\mathbb{Z}, [0, 2], [0, 0]\}$ is Backward but not Forward complete

OBSCURITY BY INCOMPLETENESS

Failing precision means failing completeness!

Obfuscating programs is making abstract interpreters incomplete

giaco
CONFIDENTIALgiaco
CONFIDENTIAL

Let $\rho \in uco(\Sigma)$ with Σ semantic objects (data, traces etc)



A program transformation $\tau: \mathbb{P} \rightarrow \mathbb{P}: \llbracket P \rrbracket = \llbracket \tau(P) \rrbracket$.



ρ \mathcal{B} -complete for $\llbracket \cdot \rrbracket$ if $\rho(\llbracket P \rrbracket) = \llbracket P \rrbracket^\rho$

giaco
CONFIDENTIAL

τ obfuscates P if $\llbracket P \rrbracket^\rho \sqsubseteq \llbracket \tau(P) \rrbracket^\rho$

giaco
CONFIDENTIAL

$$\llbracket P \rrbracket^\rho \sqsubseteq \llbracket \tau(P) \rrbracket^\rho \iff \rho(\llbracket \tau(P) \rrbracket) \sqsubseteq \llbracket \tau(P) \rrbracket^\rho$$

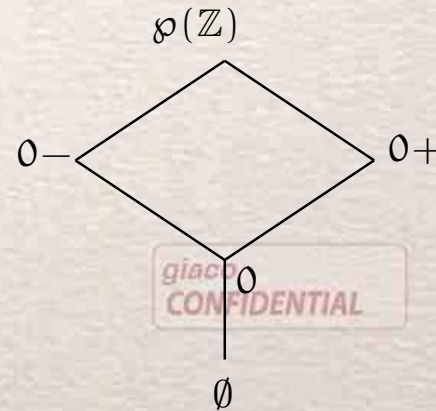
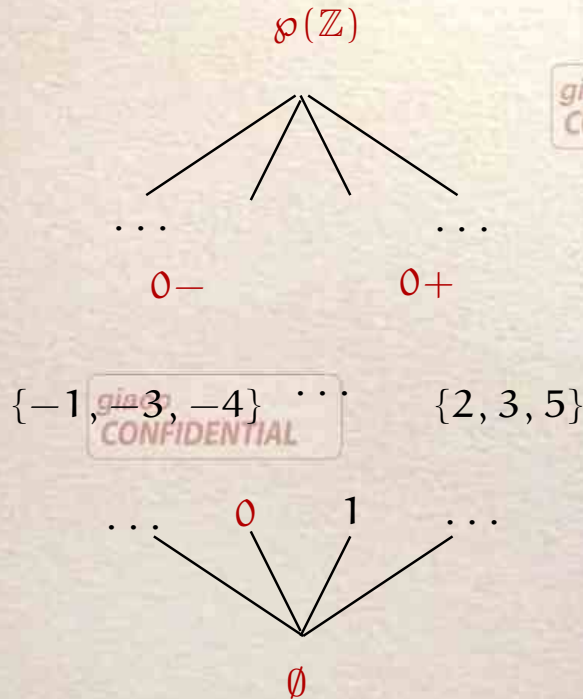
OBSCURITY BY INCOMPLETENESS

Failing precision means failing completeness!

Obfuscating programs is making abstract interpreters incomplete

$P : x = a * b$

Sign is an obvious abstraction of $\wp(\mathbb{Z})$:



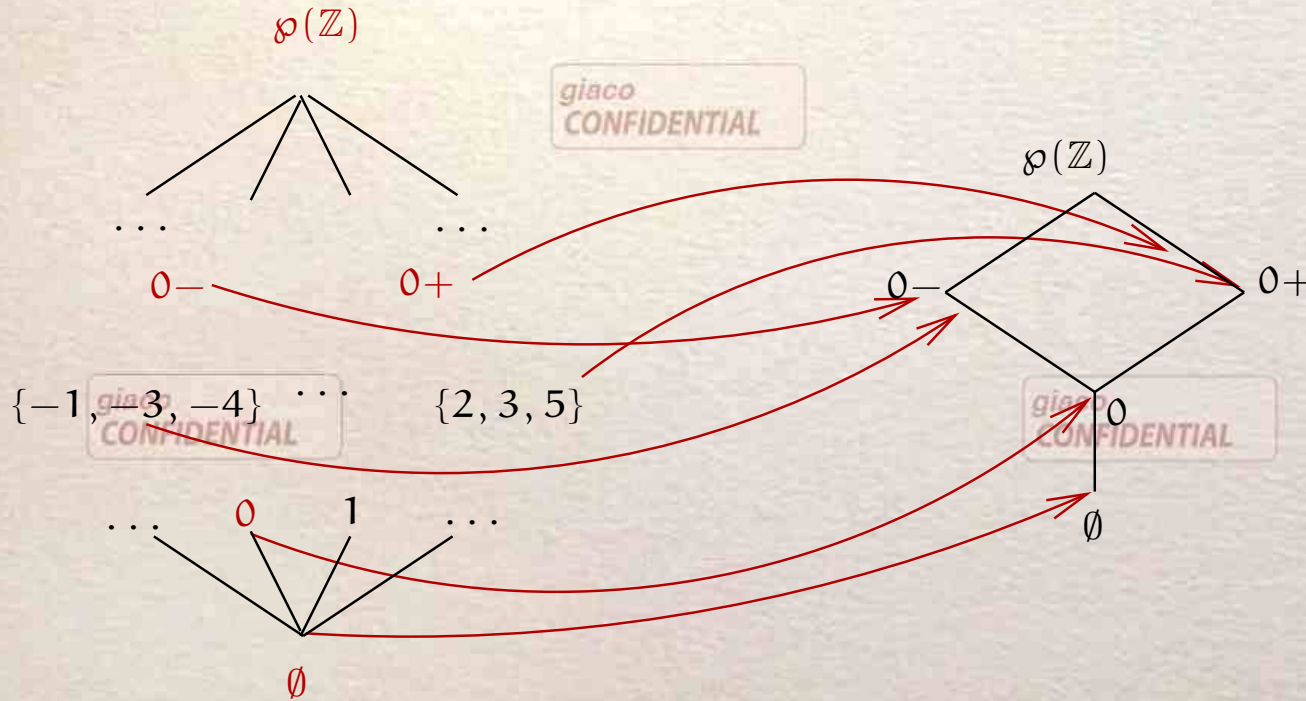
OBSCURITY BY INCOMPLETENESS

Failing precision means failing completeness!

Obfuscating programs is making abstract interpreters incomplete

$P : x = a * b$

Sign is an abstraction of $\wp(\mathbb{Z})$:



OBSCURITY BY INCOMPLETENESS

Failing precision means failing completeness!

Obfuscating programs is making abstract interpreters incomplete

giaco
CONFIDENTIALgiaco
CONFIDENTIAL

$x = 0;$
 $P: x = a * b \longrightarrow \tau(P):$ if $b \leq 0$ then $\{a = -a; b = -b\};$
 while $b \neq 0 \{x = a + x; b = b - 1\}$

giaco
CONFIDENTIAL

Sign is complete for P :

✓ $\llbracket P \rrbracket^{\text{Sign}} = \lambda a, b. \text{Sign}(a * b)$



Sign is incomplete for $\tau(P)$:

✓ $\llbracket \tau(P) \rrbracket^{\text{Sign}} = \lambda a, b. \begin{cases} 0 & \text{if } a = 0 \vee b = 0 \\ \wp(\mathbb{Z}) & \text{otherwise} \end{cases}$

giaco
CONFIDENTIAL

Is there any way to get $\tau(P)$ systematically out of P ?

GENERALIZING DATA-REFINEMENT I

We consider variable splitting:

$v \in \text{Var}(P)$ is split into $\langle v_1, v_2 \rangle$ such that
 $v_1 = f_1(v)$, $v_2 = f_2(v)$ and $v = g(v_1, v_2)$

$$\begin{aligned}f_1(v) &= v \div 10 \\f_2(v) &= v \bmod 10 \\g(v_1, v_2) &= 10 \cdot v_1 + v_2\end{aligned}$$

And the interval analysis: $\iota(x) = [\min(x), \max(x)]$

$$P : \left[\begin{array}{l} v = 0; \\ \mathbf{while} \ v < N \ \{v ++\} \end{array} \right] \quad \llbracket P \rrbracket^{\iota} = \lambda v. [0, N]$$

GENERALIZING DATA-REFINEMENT I

We consider variable splitting:

$v \in \text{Var}(P)$ is split into $\langle v_1, v_2 \rangle$ such that
 $v_1 = f_1(v)$, $v_2 = f_2(v)$ and $v = g(v_1, v_2)$

giaco
CONFIDENTIALgiaco
CONFIDENTIAL

$$f_1(v) = v \div 10$$

$$f_2(v) = v \bmod 10$$

$$g(v_1, v_2) = 10 \cdot v_1 + v_2$$

And the interval analysis: $\iota(x) = [\min(x), \max(x)]$

giaco

$$\tau(P) : \left[\begin{array}{l} v_1 = 0; \\ v_2 = 0; \\ \text{while } 10 \cdot v_1 + v_2 < N \{ \\ \quad v_1 = v_1 + (v_2 + 1) \div 10 \\ \quad v_2 = (v_2 + 1) \bmod 10 \\ \quad \}; \\ c : v = 10 \cdot v_1 + v_2 \end{array} \right.$$

$$\begin{array}{l} [[\tau(P); c]]^\iota \\ \lambda v. 10 \odot [0, \frac{N \ominus [0, 9]}{10}] \oplus [0, 9] \\ \lambda v. [0, N] \oplus [0, 9] \\ \lambda v. [0, N+9] \end{array} =$$

Obfuscation induces errors

GENERALISING DATA-REFINEMENT II

We consider **array splitting** for weakening the invariant of Fibonacci's

$$\mathbf{Inv} = 2 \leq i \leq N \wedge \forall j \in [2, i]. a[j] = a[j-1] + a[j-2]$$

The invariant **Inv** can be generated by relational interval-**Fib** analysis



$\eta = \alpha^+ \circ \alpha$ where



$$\alpha(X) = \begin{cases} \mathbf{Fib} & \text{if } \forall \langle S, x \rangle \in X. S \subseteq D_x \wedge ((S = \{0\} \wedge x[0] = 0) \vee \\ & (S = \{0, 1\} \wedge x[0] = 0 \wedge x[1] = 1) \vee \\ & (\forall j \in S. x[j] = x[j-1] + x[j-2])) \\ \mathbf{Any} & \text{otherwise} \end{cases}$$



$I \longrightarrow \mathbf{Fib}$ represents Fibonacci's sequences until $\max(I)$



$I \longrightarrow \mathbf{Any}$ represents any array with domain including I (no overflow)



$[n, m] \longrightarrow \mathbf{Fib} = [n, m-1] \longrightarrow \mathbf{Fib} \oplus [n, m-2] \longrightarrow \mathbf{Fib}$

GENERALISING DATA-REFINEMENT II

We consider **array splitting** for weakening the invariant of Fibonacci's

$$\mathbf{Inv} = 2 \leq i \leq N \wedge \forall j \in [2, i]. a[j] = a[j - 1] + a[j - 2]$$

giaco
CONFIDENTIAL

```

P :
  a[0] = 0;
  a[1] = 1;
  i = 2;
  while i ≤ N {
    a[i] = a[i - 1] + a[i - 2];
    i ++
  }

```

giaco
CONFIDENTIALgiaco
CONFIDENTIAL

$$\llbracket P \rrbracket^u \rightarrow \eta = a \in [0, N] \longrightarrow \mathbf{Fib} \wedge i \in [2, N + 1]$$

giaco
CONFIDENTIAL

GENERALISING DATA-REFINEMENT II

We consider **array splitting** for weakening the invariant of Fibonacci's

$$\mathbf{Inv} = 2 \leq i \leq N \wedge \forall j \in [2, i]. a[j] = a[j - 1] + a[j - 2]$$

```

giaco CONFIDENTIAL
τ(P) :
  b[0] = 0;
  c[0] = 1;
  i = 2;
  while i ≤ N {
    if i mod 2 == 0
      { b[i ÷ 2] = c[(i - 1) ÷ 2] + b[(i - 2) ÷ 2] }
      { c[i ÷ 2] = b[(i) ÷ 2] + c[(i - 2) ÷ 2] };
      i ++
  }
giaco CONFIDENTIAL

```

$$\llbracket \tau(P) \rrbracket \stackrel{u \longrightarrow \eta}{=} b, c \in [0, N \div 2] \longrightarrow \mathbf{Any} \wedge i \in [2, N + 1]$$

GENERALISING DATA-REFINEMENT II

We consider **array splitting** for weakening the invariant of Fibonacci's

$$\mathbf{Inv} = 2 \leq i \leq N \wedge \forall j \in [2, i]. a[j] = a[j - 1] + a[j - 2]$$

The analysis of $\tau(P)$ is unable to get **Inv** back!!

giaco
CONFIDENTIAL

giaco
CONFIDENTIAL

giaco
CONFIDENTIAL

STANDARD OBFUSCATION

1. **Abstraction transformations:** Destroy module structure, classes, functions, etc.!
2. **Data transformations:** Replace data structures with new representations!
3. **Control transformations:** Destroy if-, while-, repeat-, etc.!
4. **Dynamic transformations:** Make the program change at runtime!

CONFIDENTIAL

giaco
CONFIDENTIAL

giaco
CONFIDENTIAL

giaco
CONFIDENTIAL

giaco
CONFIDENTIAL

giaco
CONFIDENTIAL

giaco
CONFIDENTIAL

giaco
CONFIDENTIAL

OPAQUE PREDICATES



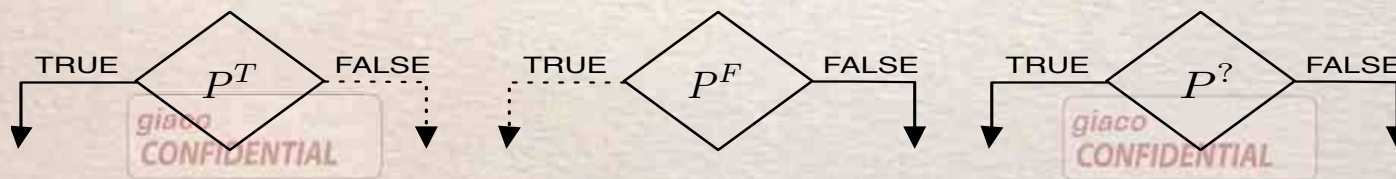
Simply put:

an expression whose value is known to you as the defender (at obfuscation time) but which is difficult for an attacker to figure out



Notation:

- ✓ P^T for an opaquely true predicate
- ✓ P^F for an opaquely false predicate
- ✓ $P^?$ for an unknown predicate



OPAQUE PREDICATES

Examples of opaque predicates from number theory

$$\forall x, y \in \mathbb{Z} : 7y^2 - 1 \neq x^2$$

giaco
CONFIDENTIAL

$$\forall x \in \mathbb{Z} : 2 \mid (x + x^2)$$

giaco
CONFIDENTIAL

$$\forall x \in \mathbb{Z} : 3 \mid (x^3 - x)$$

$$\forall n \in \mathbb{Z}^+, x, y \in \mathbb{Z} : (x - y) \mid (x^n - y^n)$$

$$\forall n \in \mathbb{Z}^+, x, y \in \mathbb{Z} : 2 \mid n \vee (x + y) \mid (x^n + y^n)$$

giaco
CONFIDENTIAL

$$\forall n \in \mathbb{Z}^+, x, y \in \mathbb{Z} : 2 \nmid n \vee (x + y) \mid (x^n - y^n)$$

$$\forall x \in \mathbb{Z}^+ : 9 \mid (10^x + 3 \cdot 4^{(x+2)} + 5)$$

$$\forall x \in \mathbb{Z} : 3 \mid (7x - 5) \Rightarrow 9 \mid (28x^2 - 13x - 5)$$

giaco
CONFIDENTIAL

$$\forall x \in \mathbb{Z} : 5 \mid (2x - 1) \Rightarrow 25 \mid (14x^2 - 19x - 19)$$

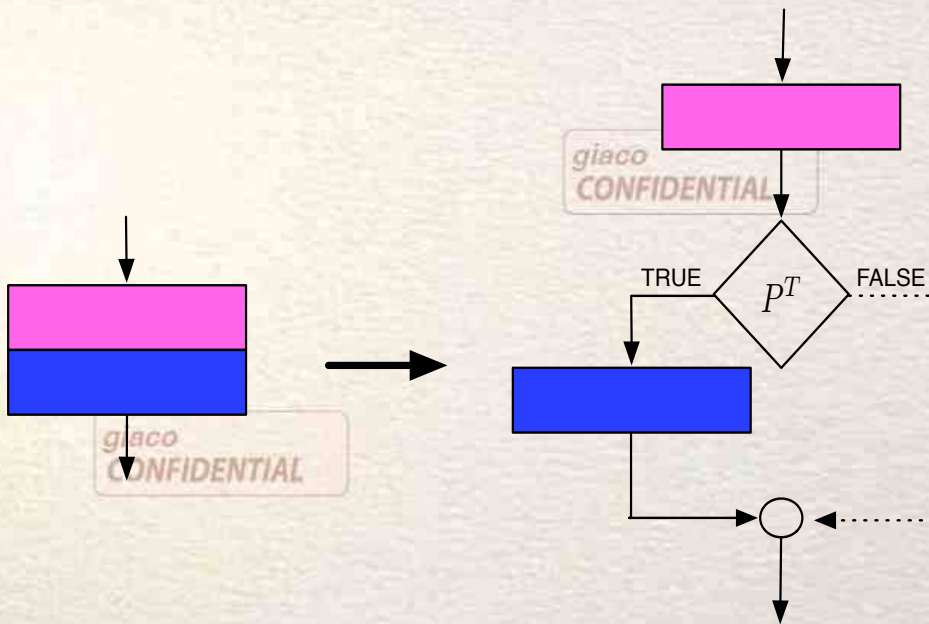
giaco
CONFIDENTIAL

$$\forall x, y, z \in \mathbb{Z} : (2 \nmid x \wedge 2 \nmid y) \Rightarrow x^2 + y^2 \neq z^2$$

$$\forall x \in \mathbb{Z}^+ : 14 \mid (3 \cdot 7^{4x+2} + 5 \cdot 4^{2x-1} - 5)$$

OPAQUE PREDICATE INSERTION

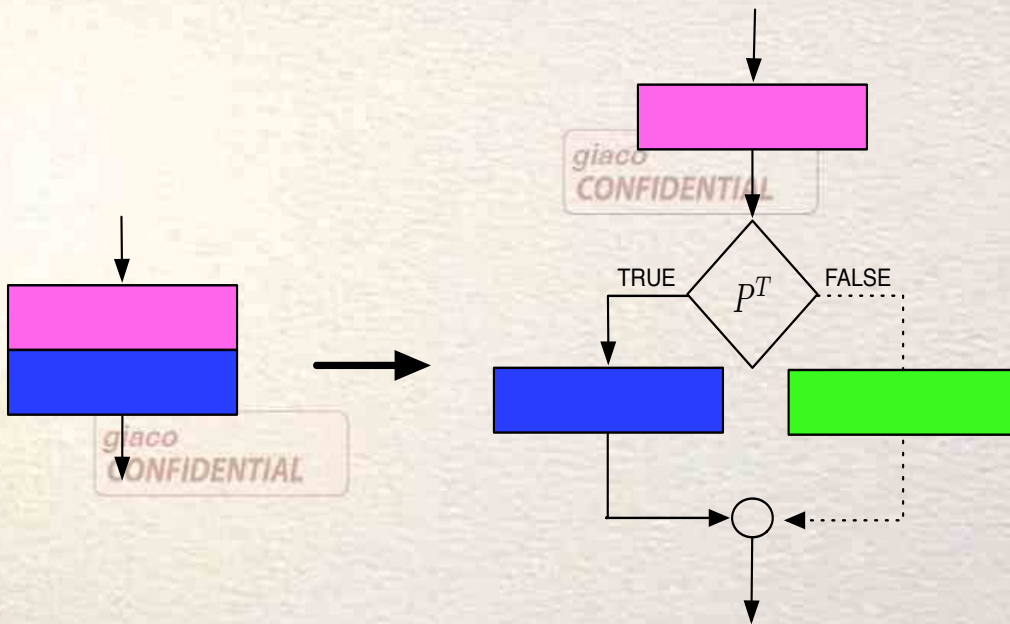
The simplest block splitting!



Attack:
Crack the predicate P^T

OPAQUE PREDICATE INSERTION

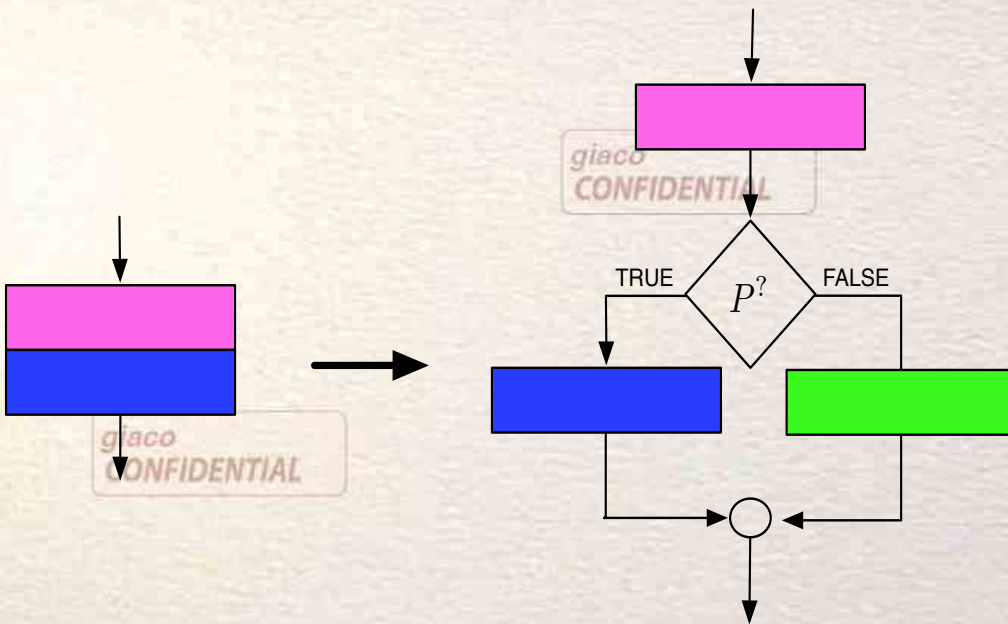
The green block can be bogus!



Attack:
Crack the predicate P^T

OPAQUE PREDICATE INSERTION

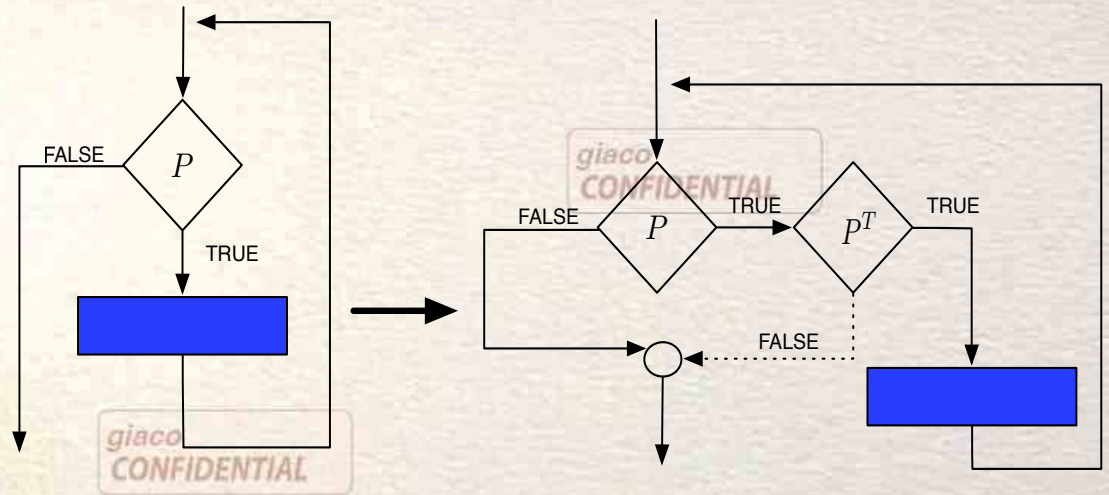
The blue and green blocks should be semantically equivalent!



Attack:
Analyse semantic equivalence!

OPAQUE PREDICATE INSERTION

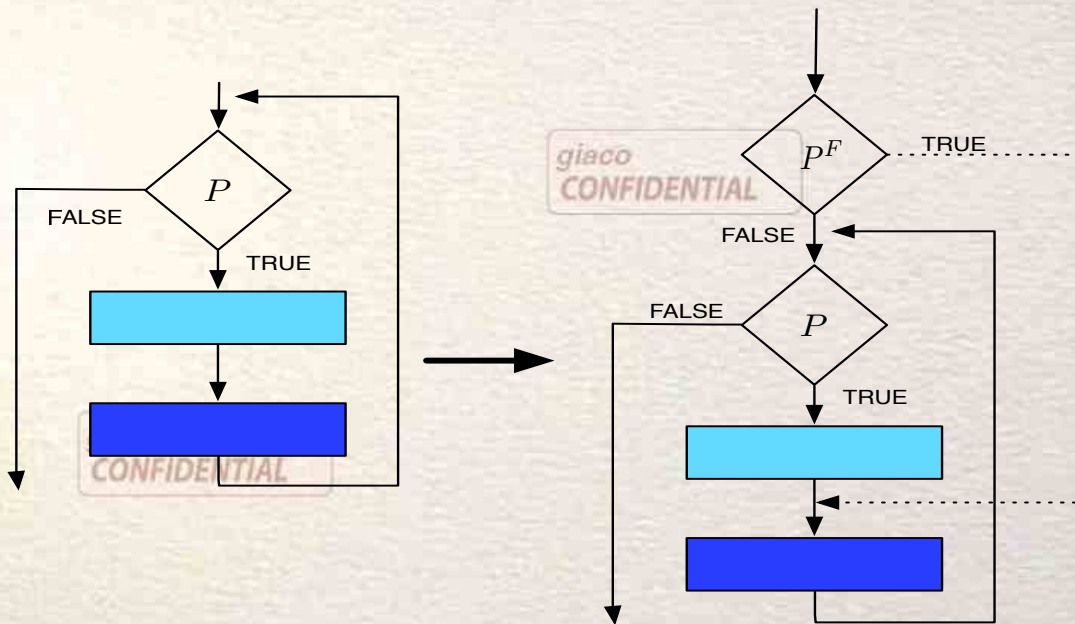
By modifying loop conditions
 P becomes $P \wedge P^T$



Attack:
Crack the predicate P^T

OPAQUE PREDICATE INSERTION

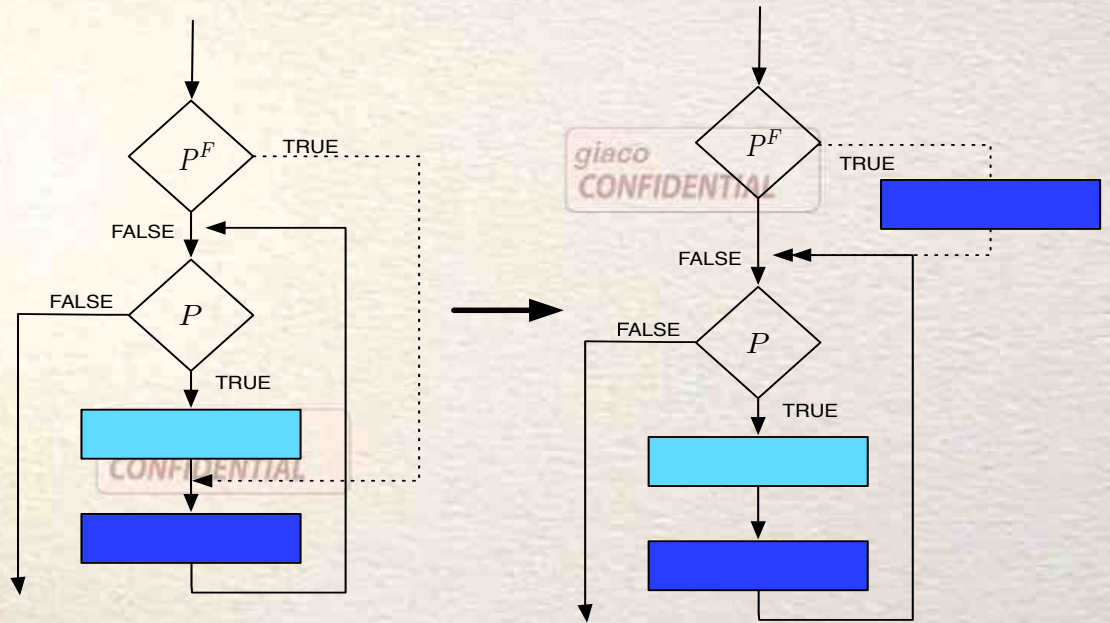
Making loops irreducible by jumping into a loop!



Attack:
Crack the predicate P^F

OPAQUE PREDICATE ATTACK

Making obfuscated loops
reducible
by *code replication*!



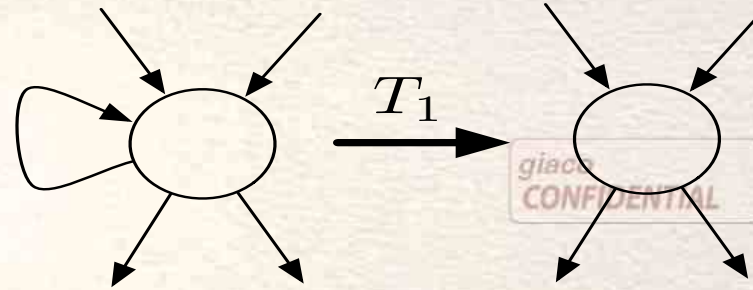
Attack:

Very hard to get due to exponential blowup!

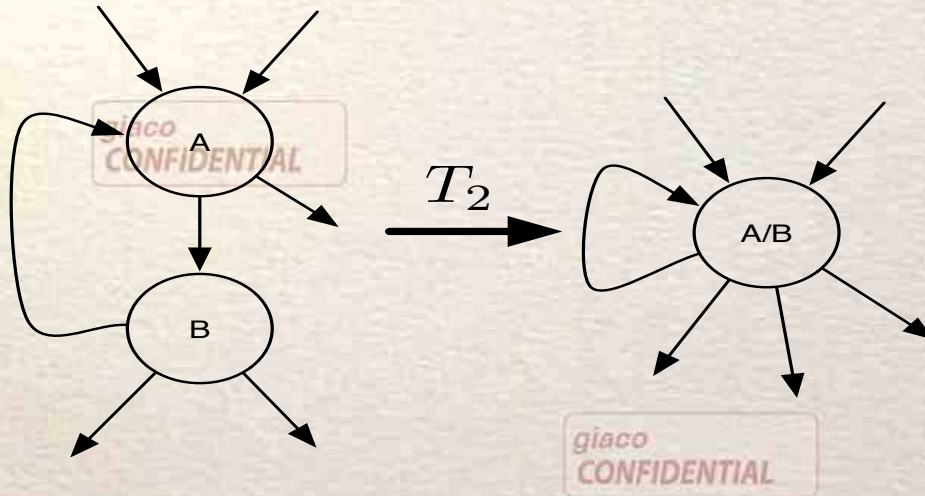
L. Carter, J. Ferrante, C. Thomborson

Folklore Confirmed: Reducible Flow Graphs are Exponentially Larger, POPL 2003

OPAQUE PREDICATE ATTACK



G is **reducible** if $G \xrightarrow{T_1/T_2} \odot$

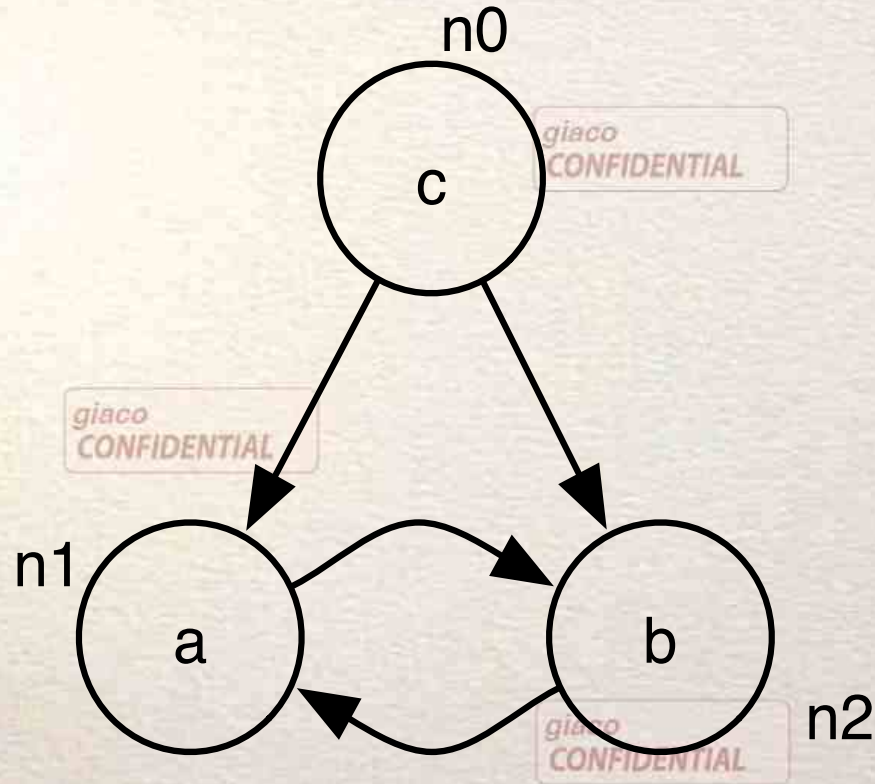


Attack:

Very hard to get due to exponential blowup!

OPAQUE PREDICATE ATTACK

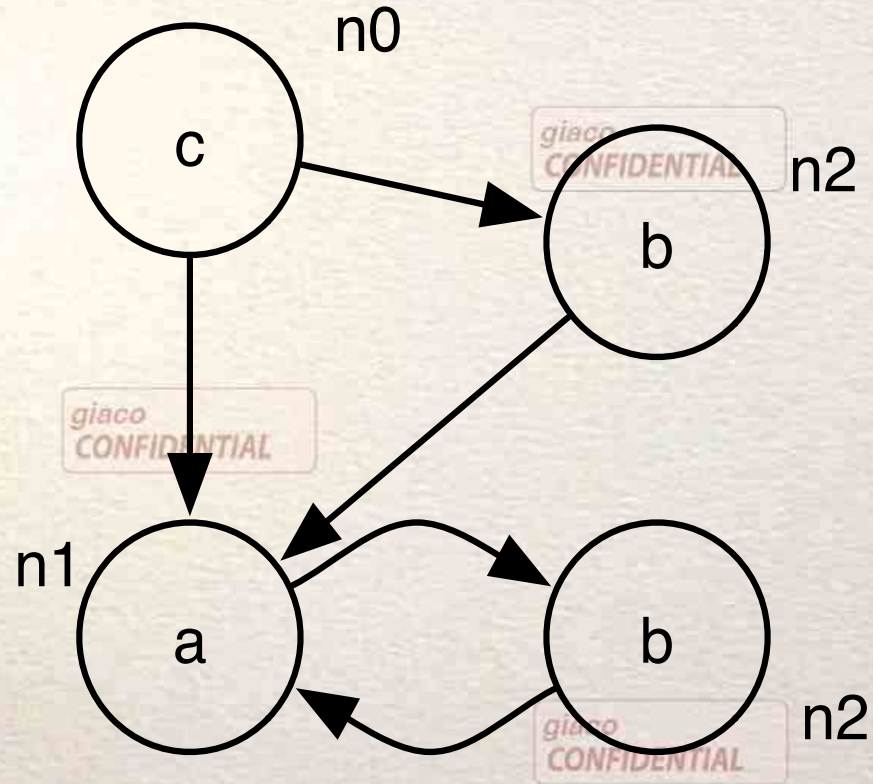
```
c: if (cond) goto b  
a: x = ...  
b: y = ...  
   goto a
```



Attack:
Typical decompiler attack!

OPAQUE PREDICATE ATTACK

```
c: if (cond) then  
    y = ...  
while (T) {  
    x = ...  
    y = ...  
}
```



Attack:
Typical decompiler attack!

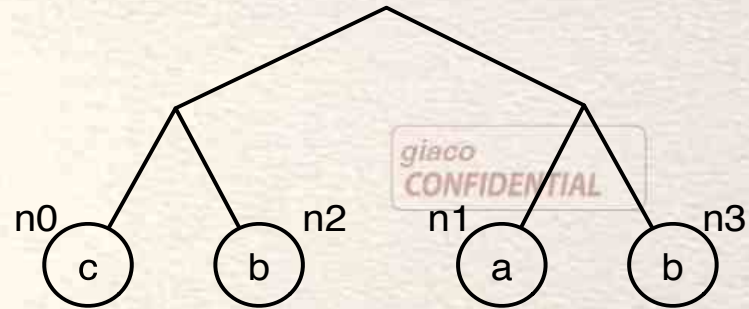
OPAQUE PREDICATE ATTACK

A **leftist tree** T is simply a rooted binary tree

$\pi = p_1 p_2 \dots p_k$ is a **leftist leaf sequence** of T if $\exists T_i$ subtree of T such that

$p_1 \in T_i$ and

$p_{i+1} = \text{left}_l(T_i) \vee p_{i+1} = \text{left}_r(T_i)$

giaco
CONFIDENTIAL

$n_0 n_2 n_2 n_1 n_0 n_1$ is ok
 $n_0 n_3$ is wrong!

Idea:

Associate a leftist tree with reducible graphs!

giaco
CONFIDENTIAL

Fact:

giaco
CONFIDENTIAL

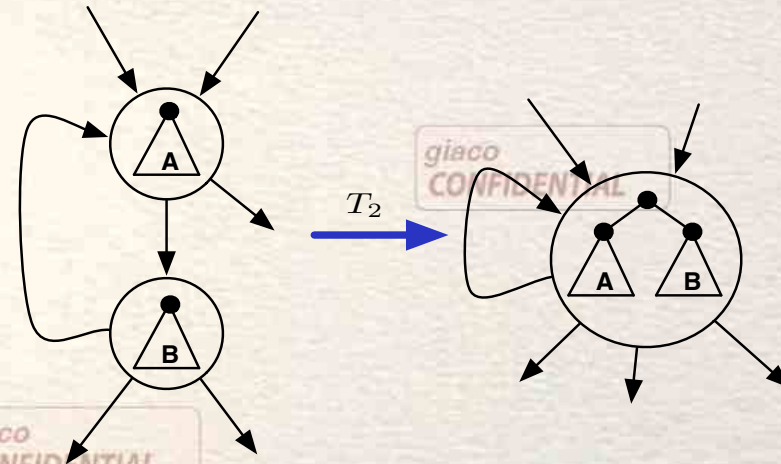
$\Gamma \subseteq \Sigma$, T is Γ -complete if $\forall \sigma \in \Gamma^* : \sigma$ is **suffix** of $\mathcal{L}(T)$

If $|\Gamma| = n$ the minimal number of leaves of a Γ -complete tree is 2^{n-1}

EXPONENTIAL BLOWUP

Theorem:

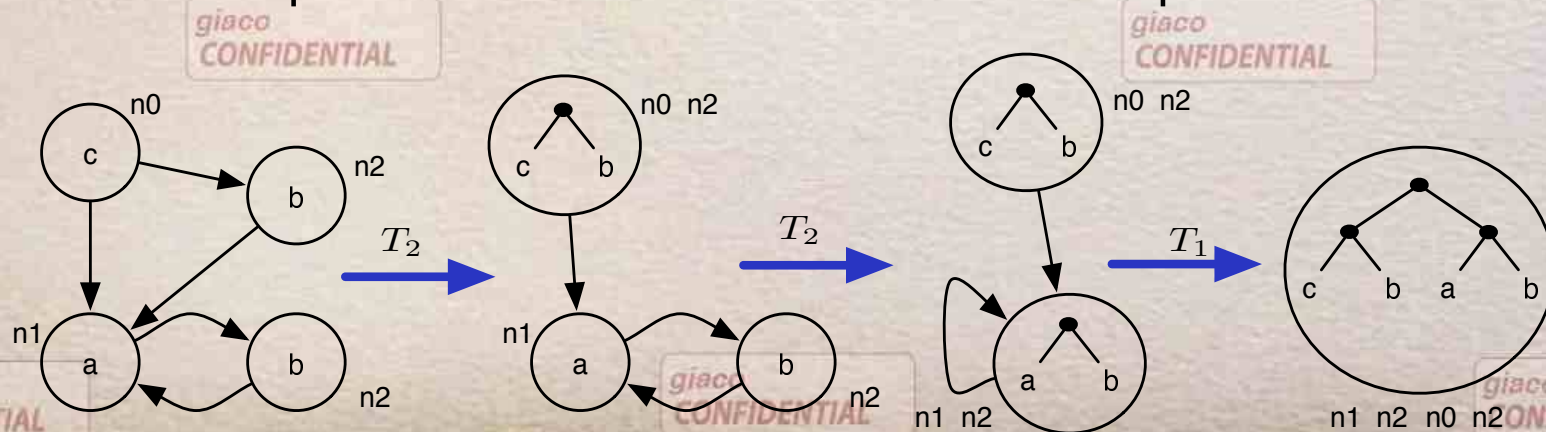
If R is a *reducible graph* equivalent to a graph with n nodes (with labels in Γ) with initial label c and $\mathcal{L}(R) = c\Gamma^*$, then R must have at least $2^{|\Gamma|-1}$ nodes.



Proof:

Associate a leftist tree with reducible graphs!

The tree corresponds to the basic transformations required for reducibility



OPAQUE PREDICATES AND EXPONENTIAL BLOWUP



Note:

complete flow graph, have no equivalent reducible flow graph that is not exponential in size



Consequence:

- ✓ No node splitting technique can avoid this exponential blowup.
- ✓ Automatic compiler analyses would find handling such large programs difficult
- ✓ **distribute code whose flow graph is irreducible!!**

⇒ **exponential number of opaque predicates!!**

L. Carter, J. Ferrante, C. Thomborson

Folklore Confirmed: Reducible Flow Graphs are Exponentially Larger, POPL 2003

OPAQUE PREDICATES FROM POINTER ALIASING



Generate opaque predicates by spurious aliases

Potency measure by complexity of static analysis



1-level aliasing is easy **P** [Banning '79]



\geq 2-level aliasing is hard **NP** [Horowitz '97]



with dynamic memory allocation is undecidable!!

understanding control-flow = solve a \geq 2-level aliasing problem

C. Collberg, C. Thomborson, D. Low

Manufacturing cheap, resilient, and stealthy opaque predicates, POPL 1998

OPAQUE PREDICATES FROM POINTER ALIASING



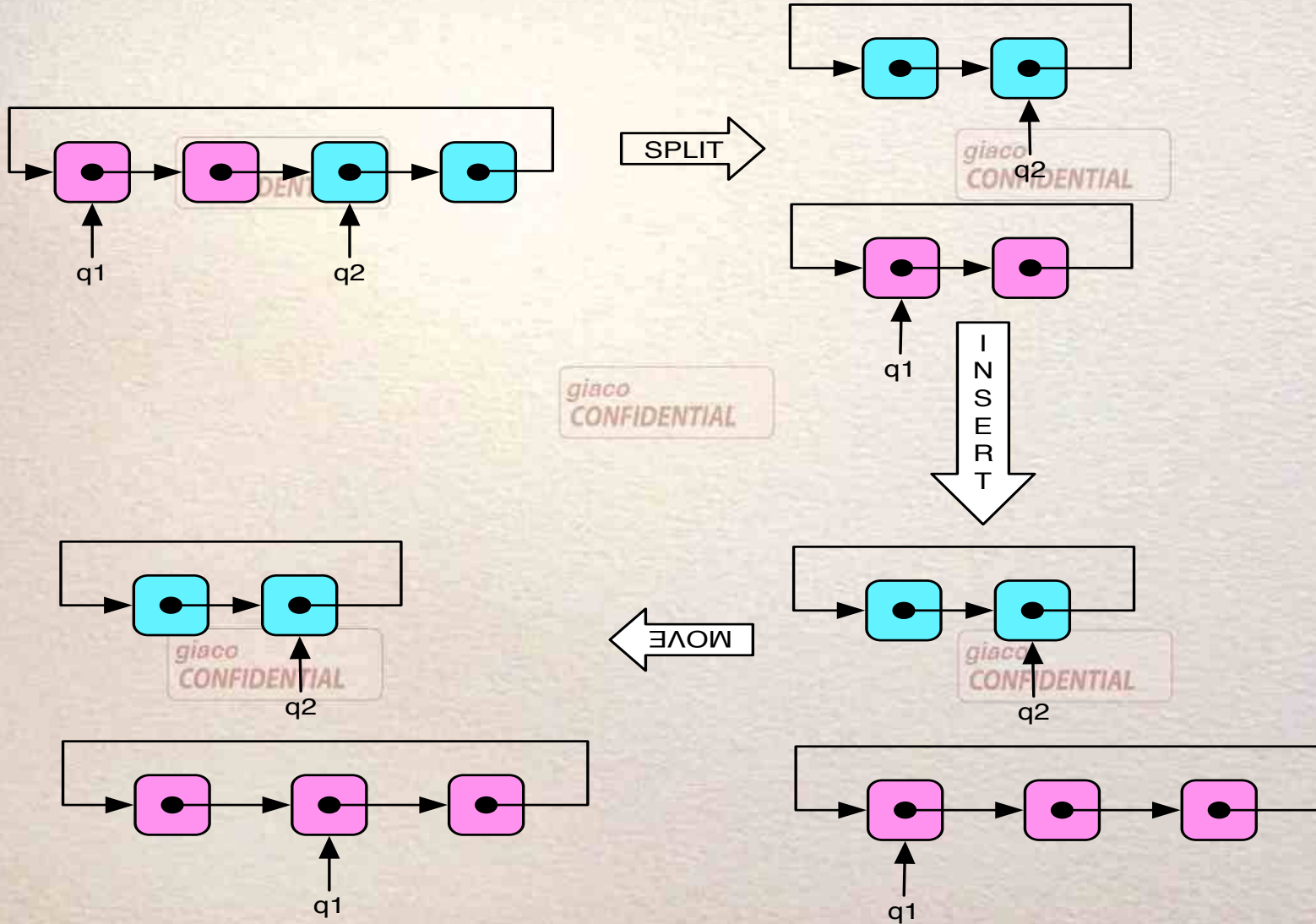
Input: A program P

1. $P := P \cup R_b$ such that R **builds** a *dynamically allocated pointer-structure* $G = \{G_1, G_2, \dots\}$
2. $P := P \cup R_p$ such that R_p add pointers $Q = \{q_1, q_2, \dots\}$ points to structures in G
3. Construct a series of invariants $I = \{I_1, I_2, \dots\}$ over G and Q such that
 - ✓ q_j always points to G_j
 - ✓ G_i is always strongly connected
4. $P := P \cup R_m$ such that R_m *occasionally* modifies the structures in G and pointers in Q while keeping invariants I
5. Using I construct opaque predicates over Q such that:
 - ✓ $(q_i \neq q_j)^T$ if q_i and q_j are known to point into different graphs
 - ✓ $(q_i \neq \text{null})^T$ if q_i is inside G_i and G_i has no leaf nodes
 - ✓ $(q_i = q_j)^?$ if q_i and q_j are known to both point into G_k

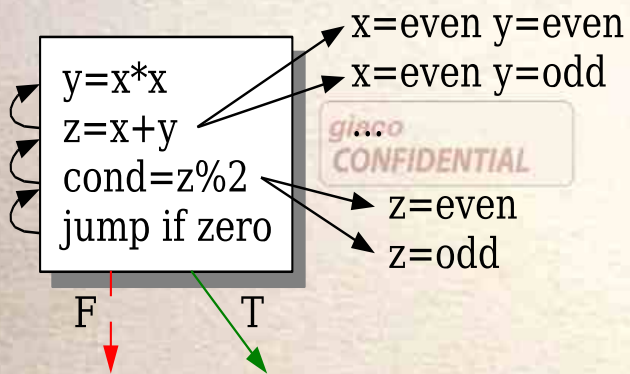
OPAQUE PREDICATES FROM POINTER ALIASING

- ⇒ Two invariants:
 1. G_1 and G_2 are circular linked lists
 2. q_1 points to a node in G_1 and q_2 points to a node in G_2 .
- ⇒ Perform enough operations to confuse even the most precise alias analysis algorithm
- ⇒ Insert opaque queries such as $(q_1 \neq q_2)^T$ into the code.

OPAQUE PREDICATES FROM POINTER ALIASING



A MEASURE OF POTENCY



Breaking $\forall x \in \mathbb{Z}, 2|(x^2 + x)$
in SPECint2000 Bench

	# opaque pred	time obf	time deobf
bzip2	442	0.53	0.13
crafty	3298	35.18	0.47
gap	6659	8.59	1.02
gcc	28006	476.33	3.11
gzip	734	0.44	0.11
mcf	189	0.29	0.06
parser	2543	2.48	0.31
perlbmk	10255	42.71	1.23
twolf	3575	1.88	0.48
vortex	8269	8.79	0.91
vpr	2206	1.44	0.3
Total	66176	578.66	8.13



The **brute force attack** took **8.83 seconds** to detect only **1** opaque predicate

$$\text{Example: } 3 \bmod (x^3 - x) = 0$$



Abstract interpretation-based de-obfuscation attack took **8.13 seconds** to de-obfuscate **66176** opaque predicates in a 32-bit env.

WHAT WE HAVE LEARNED SO FAR?

*Dalla Preda & Giacobazzi. Semantic-based Code Obfuscation by Abstract Interpretation.
J. of Computer Security 2009*

giaco
CONFIDENTIALgiaco
CONFIDENTIAL

Attacking is refining the abstract interpreter towards completeness

- ✓ A good measure of Potency
- ✓ We know how to do it!!!
 - » By abstract domain refinement
 - » By spurious counterexample elimination
 - » By Dynamic analysis

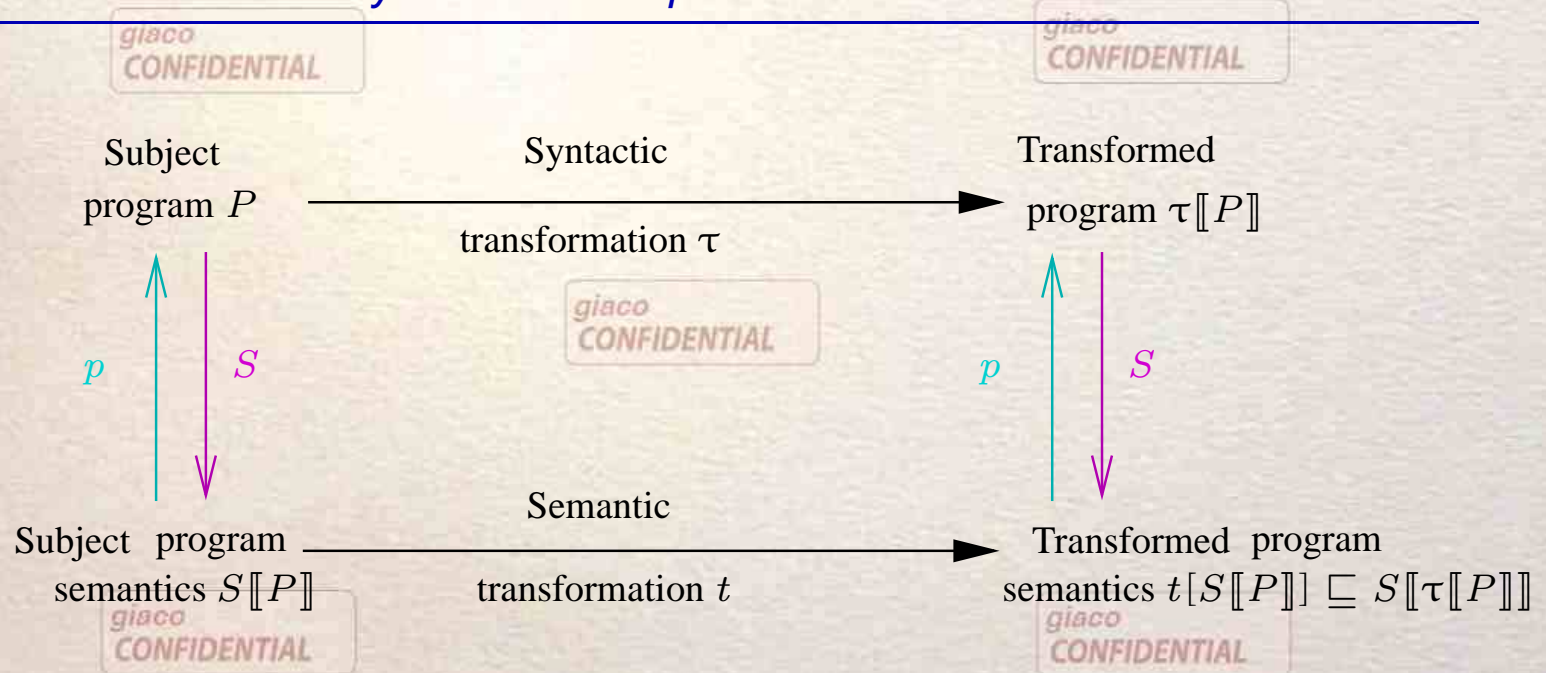
giaco
CONFIDENTIALgiaco
CONFIDENTIALgiaco
CONFIDENTIAL

Obfuscating is making (abstract) interpreters incomplete

- ✓ Is there any systematic way to do it?

OBfuscation BY PROGRAM TRANSFORMATION

Cousot & Cousot, *Systematic Design of Program Transformation Frameworks by Abstract Interpretation*. POPL'02



$$\text{Syntactic transformation: } \tau = p \circ t \circ S$$

THE GEOMETRY OF SEMANTICS TRANSFORMERS

MAKING SEMANTICS COMPLETE (FROM ABOVE AND BELOW):

$$\begin{aligned} \mathbb{F}_{\eta, \rho}^{\uparrow}(f) &= \bigsqcap \{h : C \longrightarrow C \mid f \sqsubseteq h, \rho \circ h \circ \eta = h \circ \eta\} \\ \mathbb{F}_{\eta, \rho}^{\downarrow}(f) &= \bigsqcup \{h : C \longrightarrow C \mid f \sqsupseteq h, \rho \circ h \circ \eta = h \circ \eta\} \end{aligned}$$

$\mathbb{F}_{\eta, \rho}^{\uparrow}(f)$ and $\mathbb{F}_{\eta, \rho}^{\downarrow}(f)$ are (Forward) complete

MAKING SEMANTICS MAXIMALLY IN-COMPLETE (FROM ABOVE AND BELOW):

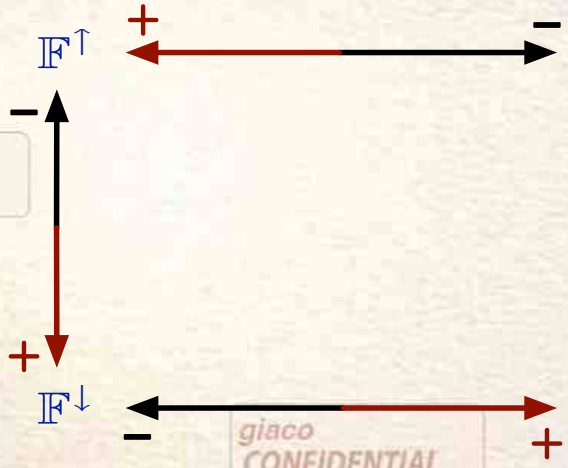
$$\begin{aligned} \mathbb{O}_{\eta, \rho}^{\uparrow}(f) &= \bigsqcup \{g : C \longrightarrow C \mid \mathbb{F}_{\eta, \rho}^{\downarrow}(g) = \mathbb{F}_{\eta, \rho}^{\downarrow}(f)\} \\ \mathbb{O}_{\eta, \rho}^{\downarrow}(f) &= \bigsqcap \{g : C \longrightarrow C \mid \mathbb{F}_{\eta, \rho}^{\uparrow}(g) = \mathbb{F}_{\eta, \rho}^{\uparrow}(f)\} \end{aligned}$$

$\mathbb{O}_{\eta, \rho}^{\uparrow}(f)$ and $\mathbb{O}_{\eta, \rho}^{\downarrow}(f)$ are generally in-complete

THE GEOMETRY OF SEMANTICS TRANSFORMERS

Minimal complete
transformation
from above

giaco
CONFIDENTIAL



Maximal incomplete
transformation
from below

giaco
CONFIDENTIAL

Minimal complete
transformation
from below

giaco
CONFIDENTIAL

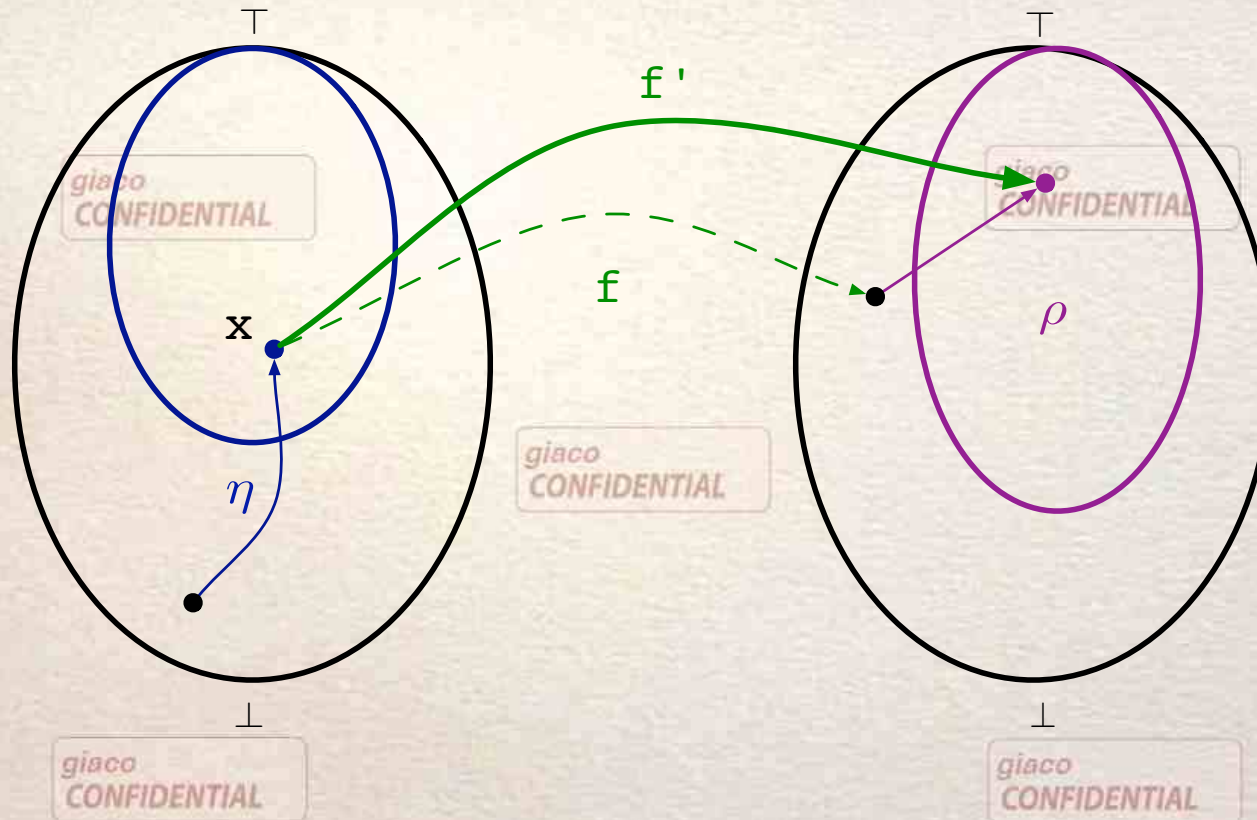
Maximal incomplete
transformation
from above

giaco
CONFIDENTIAL
 $(F^{\uparrow})^+ = F^{\downarrow}$
and
 $(F^{\uparrow})^- = O^{\downarrow}$
giaco
CONFIDENTIAL

Giacobazzi & Mastroeni, Transforming abstract interpretations by abstract interpretation

SAS'08

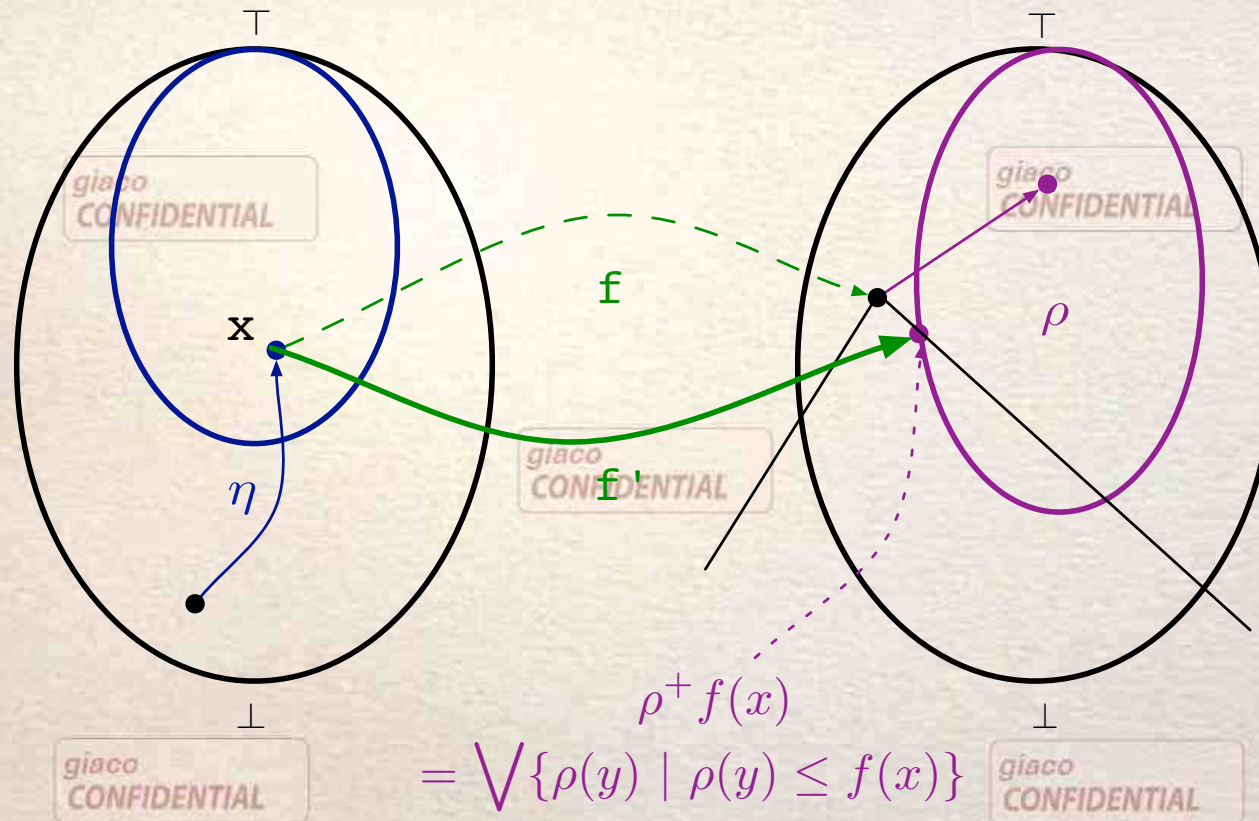
THE GEOMETRY OF SEMANTICS TRANSFORMERS



Making FORWARD COMPLETENESS: Transforming the semantics upwards

$$\mathbb{F}_{\eta, \rho}^{\uparrow} = \lambda f. \lambda x. \begin{cases} \rho \circ f(x) & \text{if } x \in \eta(C) \\ f(x) & \text{otherwise} \end{cases}$$

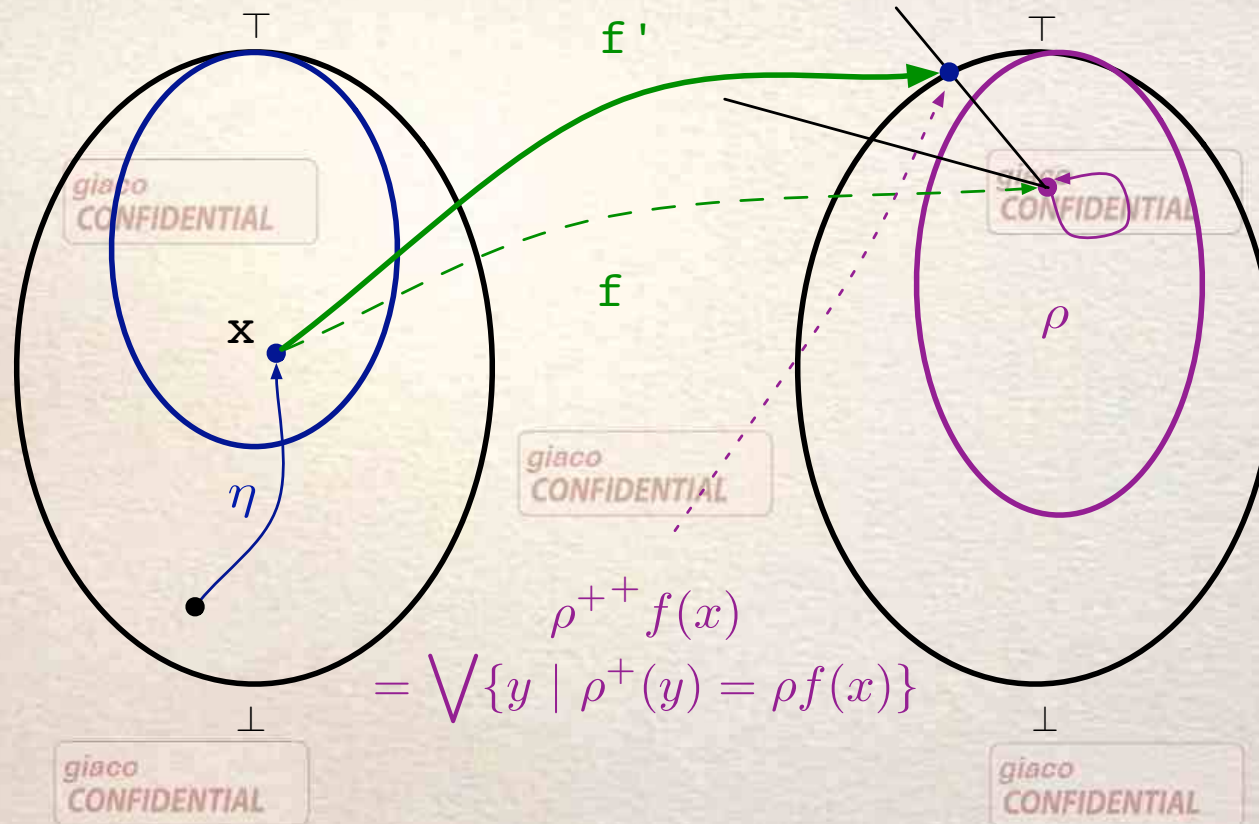
THE GEOMETRY OF SEMANTICS TRANSFORMERS



Making FORWARD COMPLETENESS: Transforming the semantics downwards

$$\mathbb{F}_{\eta, \rho}^{\downarrow} = \lambda f. \lambda x. \begin{cases} \rho^+ \circ f(x) & \text{if } x \in \eta(C) \\ f(x) & \text{otherwise} \end{cases}$$

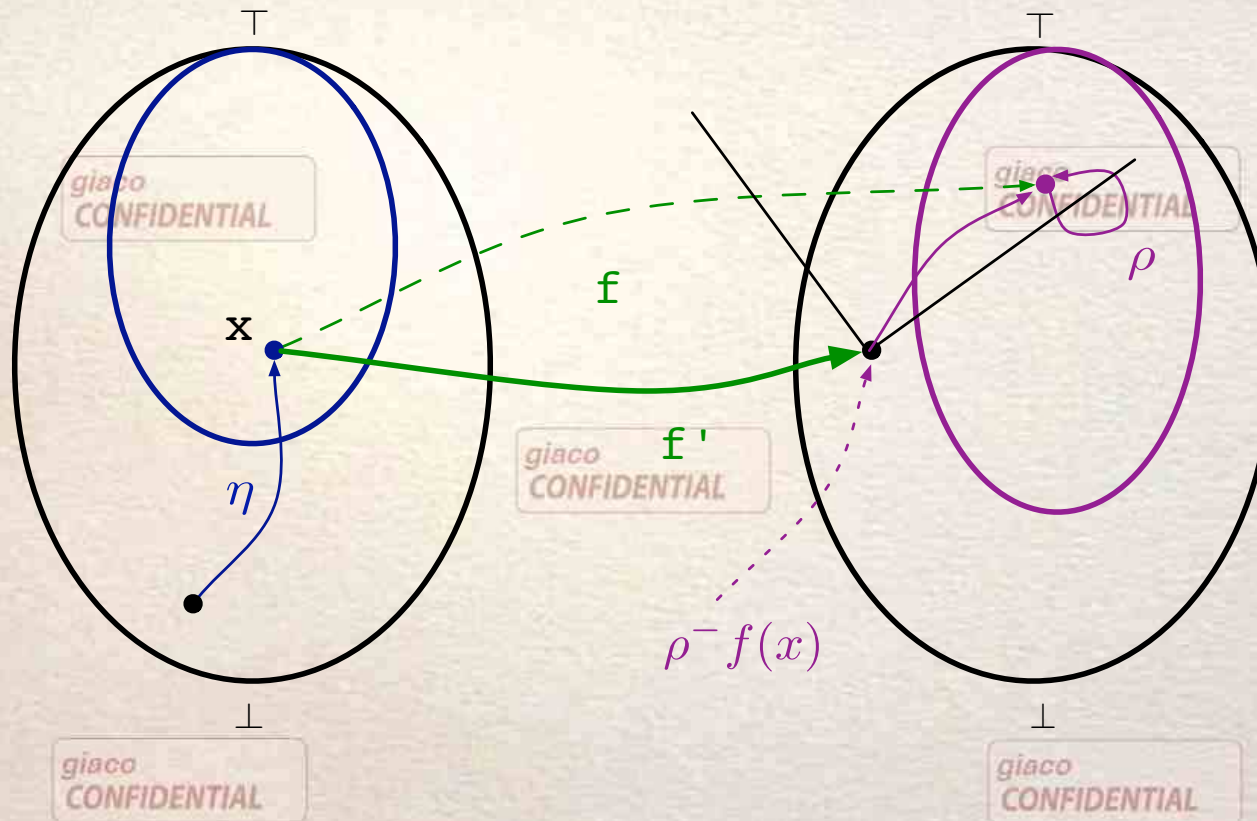
THE GEOMETRY OF SEMANTICS TRANSFORMERS



Making FORWARD IN-COMPLETENESS: Transforming the semantics upwards

$$\mathbb{O}_{\eta, \rho}^{\uparrow}(f)(x) = \begin{cases} (\rho^+)^+(f(x)) = \sqrt{\{y \mid \rho^+(y) = \rho^+(f(x))\}} & \text{if } x \in \eta \\ f(x) & \text{otherwise} \end{cases}$$

THE GEOMETRY OF SEMANTICS TRANSFORMERS



Making FORWARD IN-COMPLETENESS: Transforming the semantics downwards

$$\mathbb{O}_{\eta, \rho}^{\downarrow}(f)(x) = \begin{cases} \rho^{-}(f(x)) = \bigwedge \left\{ y \mid \rho(y) = \rho(f(x)) \right\} & \text{if } x \in \eta \\ f(x) & \text{otherwise} \end{cases}$$

WHAT DO WE HAVE SO FAR?



We know how to transform semantics to make them:

- ✓ Minimally complete
- ✓ Maximally incomplete

Incompleteness is essential for obfuscation



These are not transformations but **deformations!**

- ✓ The semantics may change
- ✓ We have to cope with this change

OBFUSCATION AS INCOMPLETENESS

We transform semantics in order to induce **maximal incompleteness**

$$P: \left[\begin{array}{l} c: \quad x = x * x; \\ \quad \text{if } 10 \leq x \leq 100 \{y = 5\} \{y = 5000\}; \\ \quad \text{return}(y) \end{array} \right.$$


$$\llbracket P \rrbracket^{\iota}(x \in [5, 8]) = x \in [25, 64] \wedge y \in [5]$$



$$\begin{aligned} \mathbf{wlp} \llbracket c \rrbracket^{\iota}(y \leq 100) &= x \in [10, 100] \text{ and} \\ \mathbf{wlp} \llbracket x = x * x \rrbracket^{\iota}(x \in [10, 100]) &= x \in [4, 10]. \end{aligned}$$



Find c' such that

$$\begin{aligned} \mathbf{wlp} \llbracket c' \rrbracket^{\iota}(x \in [10, 100]) &= \\ \mathbb{O}_{\iota, \iota}^{\downarrow}(\lambda X. \mathbf{wlp} \llbracket x = x * x \rrbracket^{\iota}(X))(x \in [10, 100]) &= \\ \iota^{-}(\mathbf{wlp} \llbracket x = x * x \rrbracket^{\iota}(x \in [10, 100])) &= \{4, 10\} \end{aligned}$$

OBFUSCATION AS INCOMPLETENESS

We transform semantics in order to induce **maximal incompleteness**

$$P: \left[\begin{array}{l} x = x * x; \\ \text{if } 10 \leq x \leq 100 \{y = 5\} \{y = 5000\}; \\ \text{return}(y) \end{array} \right.$$


$$c' : \text{if } x == 4 \vee x == 10 \{x = 16\} \{x = x * 200\}$$


In order to ensure behaviour equivalence we derive

$$\text{if } 4 \leq x \leq 10 \\ \{x = x - (x - 4) \square x = x - (x - 10)\} \\ \{\text{nil}\}$$

OBFUSCATION AS INCOMPLETENESS

We transform semantics in order to induce **maximal incompleteness**

giaco CONFIDENTIAL

$$P: \left[\begin{array}{l} c: \\ x = x * x; \\ \text{if } 10 \leq x \leq 100 \{y = 5\} \{y = 5000\}; \\ \text{return}(y) \end{array} \right.$$

giaco CONFIDENTIAL



The resulting **obfuscated code** is:

giaco CONFIDENTIAL

$$\tau(P): \left[\begin{array}{l} \text{if } 4 \leq x \leq 10 \\ \quad \{x = x - (x - 4) \square x = x - (x - 10)\} \\ \quad \{\text{nil}\}; \\ \text{if } x == 4 \vee x == 10 \{x = 16\} \{x == x * 200\}; \\ \text{if } 10 \leq x \leq 100 \{y = 5\} \{y = 5000\}; \\ \text{return}(y) \end{array} \right.$$

giaco CONFIDENTIAL

For $x = 7$ we have

$$\llbracket \tau(P) \rrbracket^v(x \in [5, 8]) = x \in [16, 1400] \wedge y \in [5, 5000]$$

OBFUSCATION AS INCOMPLETENESS

We transform semantics in order to induce **maximal incompleteness**

giaco CONFIDENTIAL

$$P : \left[\begin{array}{l} c : \\ \quad x = x * x; \\ \quad \text{if } 10 \leq x \leq 100 \{y = 5\} \{y = 5000\}; \\ \quad \text{return}(y) \end{array} \right.$$

giaco CONFIDENTIAL



The resulting obfuscated code is:

giaco CONFIDENTIAL

$$\tau(P) : \left[\begin{array}{l} \text{if } 4 \leq [5, 8] \leq 10 \\ \quad \{x = [5, 8] - ([5, 8] - 4) \square x = x - (x - 10)\} \\ \quad \{\text{nil}\}; \\ \{x \in [1, 7]\} \\ \text{if } x == 4 \vee x == 10 \{x = 16\} \{x = x * 200\}; \\ \text{if } 10 \leq x \leq 100 \{y = 5\} \{y = 5000\}; \\ \text{return}(y) \end{array} \right.$$

giaco CONFIDENTIAL

For $x = 7$ we have

$$[[\tau(P)]]^{\iota}(x \in [5, 8]) = x \in [16, 1400] \wedge y \in [5, 5000]$$

OBFUSCATION AS INCOMPLETENESS

We can derive a methodology for systematically making code obscure:



$$P = M_1; \dots; M_j; \{\Phi_j\} M_{j+1}; \dots; M_n$$



Assume the invariant Φ_j can be generated with abstract interpretation α



Find C such that:

$$\mathbf{wlp}[C]^\alpha(\Phi_j) = \mathbb{O}_{\alpha, \alpha}^{\downarrow, \uparrow}(\lambda X. \mathbf{wlp}[M_j]^\uparrow(X))(\Phi_j)$$



Adjust $C \rightsquigarrow C'$ in order to keep concrete observational (I/O) behaviour

$$C' \models \Phi_j$$



$$\tau(P) = M_1; \dots; C'; \Phi_j M_{j+1}; \dots; M_n$$

THWARTING DISASSEMBLY

- ➔ Confuse as much as possible the position of the instruction boundaries in a program
- ➔ Compare the set of instructions start addresses identified by a static disassembler and the “actual” instruction addresses encountered when the program is executed (maximize this difference)
- ➔ Self-repairing disassembly: even when a disassembly error occurs the disassembly ends up re-synchronizing with the actual instruction stream
- ➔ Delay the self-repairing process as much as possible

THWARTING DISASSEMBLY: JUNK INSERTION

⇒ Introduce disassembly errors by inserting junk bytes where the disassembler expects code

- ✓ Junk bytes are partial instructions
- ✓ Junk bytes are never executed at run time

giaco
CONFIDENTIAL

⇒ Candidate block is a Basic Block that can have junk bytes inserted before (the BB preceding ends with a direct jump or a function call)

⇒ Given a candidate block we have to determine the junk bytes to insert before it in order to delay the self-repairing process as much as possible (we insert the first k byte of a given instruction I where k maximizes the delay)

giaco
CONFIDENTIALgiaco
CONFIDENTIAL

THWARTING LINEAR SWEEP



Attack limitation: Unable to distinguish data embedded in the instruction stream



Junk bytes insertion: 26% - 30% of instructions are incorrectly disassembled (candidate blocks are too far ~ 30 instructions)



Increase the number of candidate blocks through **branch flipping**:

bccAddr

bccL

L :

jmpAddr

L : ...



L is now a candidate block!!



70% of the instructions are incorrectly disassembled

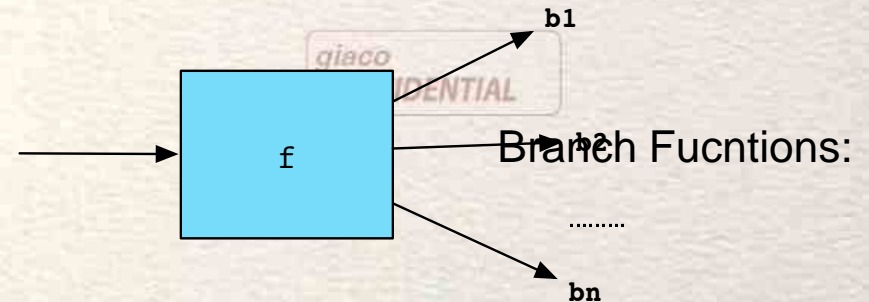
THWARTING RECURSIVE TRAVERSAL

Assumption: a function returns to the instruction following the call instruction

Branch functions:

```

a1: jmp b1      a1: call f
a2: jmp b2      a2: call f
...
an: jmp bn      an: call f
  
```



Obscure the control flow of the program



Confuse disassembly by inserting junk bytes at the points immediately after `call f`



The resulting code is more expensive (depending on the implementation of the branch functions) and therefore such transformation is not applied to “hot code” (code frequently executed)

THWARTING DISASSEMBLY: EXPERIMENTAL EVALUATION

- ➔ Identify the hot blocks
- ➔ Linear sweep: 75% of instructions are incorrectly disassembled
- ➔ Recursive traversal: 40% of instructions are incorrectly disassembled
- ➔ Average 52% execution speed penalty

DYNAMIC OBFUSCATION: DEFINITIONS

- ➔ Static obfuscations transform the code prior to execution.
- ➔ Dynamic algorithms transform the program at runtime.
- ➔ Static obfuscation counter attacks by static analysis.
- ➔ Dynamic obfuscation counter attacks by dynamic analysis
- ➔ A dynamic obfuscator runs in two phases:
 1. At compile-time transform the program to an initial configuration and add a runtime code-transformer.
 2. At runtime, intersperse the execution of the program with calls to the transformer.
- ➔ A dynamic obfuscator turns a *normal* program into a self-modifying one.

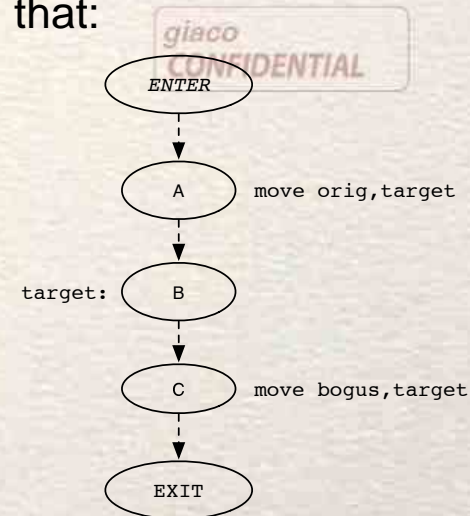
DYNAMIC OBFUSCATION: REPLACING INSTR.



Obfuscate(P):

1. Select three points A , B , and C in P , such that:

- ✓ A strictly dominates B ,
- ✓ C strictly post-dominates B , and
- ✓ any path from B to A passes through C .



2. Let `orig` be the instruction at B .
3. Select an instruction `bogus` of the same length as `orig`.
4. Replace `orig` at B with `bogus`.
5. At point A insert the instruction `move orig, $v=B$` where v is an opaque expression that evaluates to the address of point B .
6. Similarly, at point C insert the instruction `move bogus, $v=B$` .

DYNAMIC OBFUSCATION: AUCSMITH'S ALGORITHM

D. Aucsmith, Tamper resistant SW: An implementation, Patent: 5892899, 1999

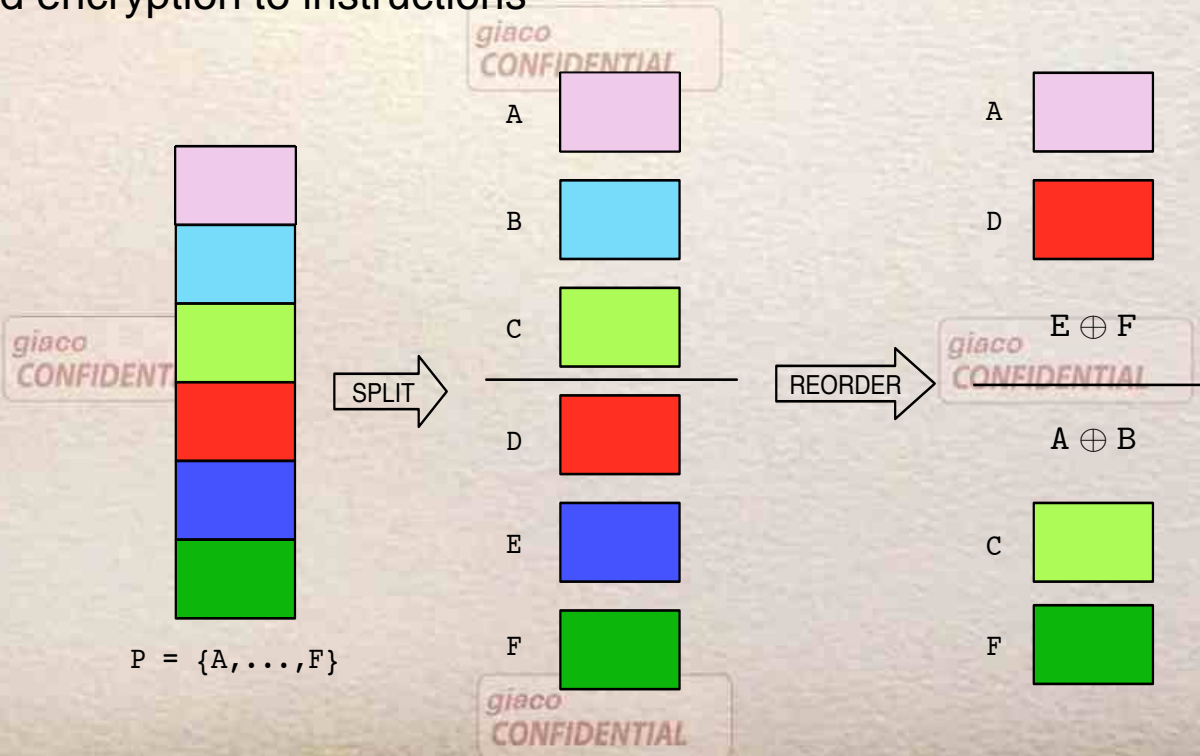


Idea:

giaco
CONFIDENTIAL

- ✓ Split the program into pieces
- ✓ XOR them with each other
- ✓ Add encryption to instructions

giaco
CONFIDENTIAL



DYNAMIC OBFUSCATION: AUCSMITH'S ALGORITHM

D. Aucsmith, Tamper resistant SW: An implementation, Patent: 5892899, 1999



In general:

- ✓ $(A \oplus B) \oplus A = B$
- ✓ $(A \oplus B) \oplus B = A$



Idea

- ✓ Reorder blocks such that upper and lower blocks are alternated
- ✓ Execute a block and `xor` upper with lower if even iterate
- ✓ Execute a block and `xor` lower with upper if odd iterate

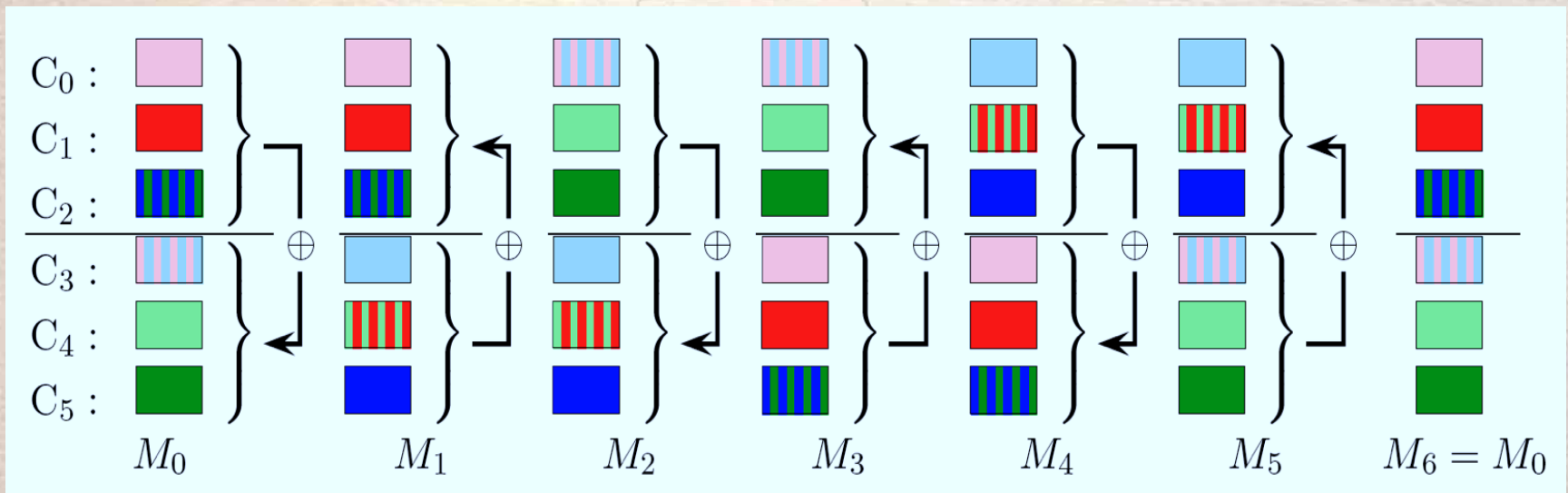
DYNAMIC OBFUSCATION: AUCSMITH'S ALGORITHM

D. Aucsmith, Tamper resistant SW: An implementation, Patent: 5892899, 1999



In general:

- ✓ $(A \oplus B) \oplus A = B$
- ✓ $(A \oplus B) \oplus B = A$



DYNAMIC OBFUSCATION: AUCSMITH'S ALGORITHM



Obfuscate(P):

1. **Split** P into n pieces: P_0, \dots, P_{n-1} .
2. Let C_0, \dots, C_{n-1} be the memory cells in which the pieces will reside at runtime. The C_i 's are of equal size, large enough to fit the largest piece P_j .
3. Cells are, conceptually, divided into two spaces, **upper** ($C_0, \dots, C_{n/2-1}$) and **lower** ($C_{n/2}, \dots, C_{n-1}$). Each cell in the upper space partners with a cell in the lower space. Select a partner function $PF(c)$ that maps a cell number to the cell number of its partner, such as $PF(c) = c + K$, for some constant K .
4. Let IV_0, \dots, IV_{n-1} be the initial values of cells C_0, \dots, C_{n-1} , respectively.
5. Initialize a set of equations $\mathcal{E}_1 = \{C_0 = IV_0, \dots, C_{n-1} = IV_{n-1}\}$ which expresses the current state of the memory cells as a function of their initial values.
6. Initialize a set of equations $\mathcal{E}_2 = \{\}$ which expresses how a piece P_i can be recovered in **cleartext** from the initial values IV_0, \dots, IV_{n-1} .
7. Initialize a table $\text{next} = \langle P_0 = ?, \dots, P_{n-1} = ? \rangle$ which maps each subprogram P_i to the cell it should jump to in order to execute P_{i+1} .
8. `make_obscure()`

DYNAMIC OBFUSCATION: AUCSMITH'S ALGORITHM



make_obscure():

- ✓ For $p \in [0 \dots n - 1]$ do
 - » Select a cell C_c to hold piece P_p in cleartext.
 - » Consult \mathcal{E}_1 to find the current contents V of C_c . Update $\mathcal{E}_2 := \mathcal{E}_2[P - p = V]$. Using *Gaussian elimination*, try to invert \mathcal{E}_2 (i.e. find values for all the IV_i 's). If there is no solution select another cell for P_p .
 - » $\text{next} := \text{next}[P_{p-1} = C_c]$
 - » For even (odd) p:s, simulate a mutation where every cell C_i in upper (lower) space is `xor`:ed with its partner cell $C_{PF(i)}$ in lower (upper) space.
 - » $\mathcal{E}_1 := \mathcal{E}_1[C_{PF(i)} = C_{PF(i)} \oplus C_i]$.

DISCUSSION: THE FUCSIA IDEA

Obfuscation and Steganography by Abstract Interpretation



Define a uniform framework for information concealment in programming languages

- ✓ **General** enough to include most known methods
- ✓ **Formal** enough to provide a (possibly) provable secure environment for obfuscation and steganography
- ✓ **Rich** enough to provide advanced design and evaluation tools
- ✓ **Practical** enough to become a standard in the obfuscation and steganographic design and evaluation



The goal: develop a theory and practice for code obfuscation and steganography in order to make these technologies as practical as analogous ones in other media (e.g., in DRM of audio and video)

- ✓ The code is a new media
- ✓ Known concepts in digital media (compression, noise etc.) have to be studied on software

FUTURE DIRECTIONS



Move from syntactic to semantic-based metrics for potency

- ✓ measuring incompleteness
- ✓ measuring complexity of complete refinements



We need a **program refinement**-like calculus for code obfuscation

- ✓ A kind of **Program Obfuscation by Stepwise Refinement** [Wirth '71] ?
- ✓ This is an open issue....