Analyzing Smart Contracts: From EVM to a sound Control-Flow Graph

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Abstract. The EVM language is a simple stack-based language with words of 256 bits, with one significant difference between the EVM and other virtual machine languages (like Java Bytecode or CLI for .Net programs): the use of the stack for saving the jump addresses instead of having it explicit in the code of the jumping instructions. Static analyzers need the complete control flow graph (CFG) of the EVM program in order to be able to represent all its execution paths. This report addresses the problem of obtaining a precise and complete stack-sensitive CFG by means of a static analysis, cloning the blocks that might be executed using different states of the execution stack. The soundness of the analysis presented is proved.

1 EVM Language

The EVM language is a simple stack-based language with words of 256 bits with a local volatile memory that behaves as a simple word-addressed array of bytes, and a persistent storage that is part of the blockchain state. A more detailed description of the language and the complete set of operation codes can be found in [6]. In this section, we focus only on the relevant characteristics of the EVM that are needed for describing our work.

Example 1. In order to describe our techniques, we use as running example a simplified version (without calls to the external service Oraclize and the authenticity proof verifier) of the contract [1] that implements a lottery system. During a game, players call a method \texttt{joinPot} to buy lottery tickets; each player’s address is appended to an array \texttt{addresses} of current players, and the number of tickets is appended to an array \texttt{slots}, both having variable length. After some time has elapsed, anyone can call \texttt{rewardWinner} which calls the Oraclize service to obtain a random number for the winning ticket. If all goes according to plan, the Oraclize service then responds by calling the __callback method with this random number and the authenticity proof as arguments. A new instance of the game is then started, and the winner is allowed to withdraw her balance using a \texttt{withdraw} method. Figure 2 shows an excerpt of the \texttt{Solidity} code (including the public function \texttt{findWinner}) and a fragment of the EVM code produced by the compiler. The \texttt{Solidity} source code is shown for readability, as our analysis works directly on the EVM code.

To the right of Figure 2 we show a fragment of the EVM code of method \texttt{findWinner}. It can be seen that the EVM has instructions for operating with the stack contents, like \texttt{DUPx} or \texttt{SWAPx}; for comparisons, like \texttt{LT}, \texttt{GT}; for accessing the storage (memory) of the contract, like \texttt{SSTORE}, \texttt{SLOAD} (\texttt{MLOAD}, \texttt{MSTORE}); to add/remove elements to/from the stack, like \texttt{PUSHx}/\texttt{POP}; and many others (we again refer to [6] for details). Some instructions increment the program counter in several units (e.g., \texttt{PUSHx Y} adds a word with the constant \(Y\) of \(x\) bytes to the stack and increments the program counter by \(x + 1\)). In what follows, we use \texttt{size(b)} to refer to the number of units that instruction \(b\) increments the value of the program counter. For instance \(\text{size(Pop)} = 1, \text{size(Push1)} = 2\) or \(\text{size(Push3)} = 4\).

One significant difference between the EVM and other virtual machine languages (like Java Bytecode or CLI for .Net programs) is the use of the stack for saving the jump addresses instead of having it explicit in the code of the jumping instructions. In EVM, instructions \texttt{JUMP} and \texttt{JUMPI} will jump, unconditionally and conditionally respectively, to the program counter stored in the top of the execution stack. This feature of the EVM requires, in order to obtain the control flow graph of the program, to keep track of the information stored in the stack. Let us illustrate it with an example.

Example 2. In the EVM code to the right of Figure 2 we can see two jump instructions at program points 953 and 660, respectively, and the jump address (\texttt{64B} and \texttt{6DO}) is stored in the instruction immediately before them: \texttt{950} or \texttt{65D}. It then jumps to this destination by using the instruction \texttt{JUMPDEST} (program points 941, 64B, 653).
We start our analysis by defining the set $J$, which contains all possible jump destinations in an EVM program $P = b_0, \ldots, b_p$:

$$J(P) = \{pc \mid b_{pc} \in P \land b_{pc} \equiv \text{JUMPDEST}\}.$$

We use $b_{pc}$ to refer to the instruction at program counter $pc$ in the EVM program $P$. In what follows, we omit $P$ from definitions when it is clear from the context, e.g., we use $J$ to refer to $J(P)$.

Example 3. Given the EVM code that corresponds to function `findWinner`, we get the following set:

$$J = \{123, 142, 954, 64B, 6D0, 66F, 653, 6C3, 691, 6D1, 6BA\}$$

The first step in the computation of the CFG is to define the notion of block. In general [2], given a program $P$, a block is a maximal sequence of straight-line consecutive code in the program with the properties that the flow of control can only enter the block through the first instruction in the block, and can only leave the block at the last instruction. Let us define the concept of block in an EVM program:

Definition 1 (blocks). Given an EVM program $P = b_0, \ldots, b_p$, we define

$$\text{blocks}(P) = \left\{ B_i \equiv b_i, \ldots, b_j \mid (\forall k, i < k < j, b_k \notin \text{Jump} \cup \text{End} \cup \{\text{JUMPDEST}\}) \land (i = 0 \lor b_i \equiv \text{JUMPDEST} \lor b_{i+1} \equiv \text{JUMP}) \land (j = p \lor b_j \equiv \text{Jump} \lor b_j \equiv \text{End} \lor b_{j+1} \equiv \text{JUMPDEST}) \right\}$$

where

\[\text{Jump} = \{\text{JUMP}, \text{JUMPI}\}\]
\[\text{End} = \{\text{REVERT}, \text{STOP}, \text{INVALID}\}\]

Example 4. Figure 3 shows the blocks (nodes) obtained for `findWinner` and their corresponding jump invocations. Solid and dashed edges represent the two possible execution paths depending on the entry block: solid edges represent the path that starts from block 941 and dashed edges the path that starts from 123. Note that most of the blocks start with a JUMPDEST instruction (123, 941, 64B, 653, 66F, 954,
EVM programs which allows us to compute a complete CFG of the block. In other tools \cite{4,3,5}, to precisely infer the possible addresses at jumping program points, we need a context-sensitive static analysis that analyze separately all blocks for each possible stack than can reach them (only considering the addresses stored in the stack). This section presents an address analysis of EVM programs which allows us to compute a complete CFG of the EVM code. To compute the addresses involved in the jumping instructions, we define a static analysis which soundly infers all possible addresses that a JUMP instruction could use.

In our address analysis we aim at having the stack represented by explicit variables. Given the characteristics of EVM programs, the execution stack of EVM programs produced from Solidity programs...
The abstract state. The abstract state is a partial mapping \( \pi \) of the form \( S \mapsto P(S) \).

The application of \( \pi \) to an element \( s_i \), that is, \( \pi(s_i) \), corresponds to the set of jump destinations that a stack variable \( s_i \) can contain. The first element of the tuple, that is, \( n \), stores the size of the stack in the different abstract states.

The abstract domain is the lattice \( AS, \pi_+, \pi_-, \cup, \subseteq \), where \( AS \) is the set of abstract states and \( \pi_+ \) is the top of the lattice defined as the mapping \( \pi_T \) such that \( \forall s \in S, \pi_T(s) = S \). The bottom element of the lattice \( \pi_- \) is the empty mapping. Now, to define \( \cup \) and \( \subseteq \), we first define the function \( img(\pi, s) \) as \( \pi(s) \) if \( s \in dom(\pi) \) and \( \emptyset \), otherwise. Given two abstract states \( \pi_1 \) and \( \pi_2 \), we use \( \pi = \pi_1 \cup \pi_2 \) to denote that \( \pi \) is the least upper-bound defined as follows \( \forall s \in dom(\pi_1) \cup dom(\pi_2), \pi(s) = img(\pi_1, s) \cup img(\pi_2, s) \). At this point, \( \pi_1 \subseteq \pi_2 \) holds iff \( dom(\pi_1) \subseteq dom(\pi_2) \) and \( \forall s \in dom(\pi_1), \pi_1(s) \subseteq \pi_2(s) \).

Transfer function. One of the ingredients of our analysis is a transfer function that models the effect of each EVM instruction on the abstract state for the different instructions. Given a stack state \( s \) of the form \( (n, \pi) \), Figure 4 defines the updating function \( \lambda(b_{pc}, s) \) where \( b \) corresponds to the EVM instruction to be applied, \( pc \) corresponds to the program counter of the instruction and \( \alpha \) and \( \delta \) to the number of elements placed to and removed from the EVM stack when executing \( b \), respectively. Given a map \( m \) we will use \( m[x \mapsto y] \) to indicate the result of updating \( m \) by making \( m(x) = y \) while \( m \) stays the same for
all locations different from \( x \), and we will use \( m \setminus \{ x \} \) to refer to a partial mapping that stays the same for all locations different from \( x \), and \( m(x) \) is undefined.

By means of \( \lambda \), now we can define the transfer function of our analysis.

**Definition 3 (transfer function).** Given the set of abstract states \( AS \) and the set of EVM instructions \( Ins \), the transfer function \( \tau \) is defined as a mapping of the form

\[
\tau : Ins \times AS \mapsto AS
\]

is defined as follows:

\[
\tau(b, \pi) = \pi' \quad \text{where } \forall s \in \text{dom}(\pi), \pi'(s) = \lambda(b, \pi(s))
\]

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PUSH ( x ) ( v )</td>
<td>( (n + 1, \sigma[s_n \mapsto { v }]) ) when ( v \in J ) ( (n + 1, \sigma) ) when ( v \notin J )</td>
</tr>
<tr>
<td>DUP ( x )</td>
<td>( (n + 1, \sigma) ) when ( s_{n-1} \notin \text{dom}(\sigma) ) ( (n + 1, \sigma[s_n \mapsto \sigma(s_{n-1})]) ) when ( s_{n-1} \in \text{dom}(\sigma) )</td>
</tr>
<tr>
<td>SWAP ( x )</td>
<td>( (n, \sigma) ) when ( s_{n-1} \notin \text{dom}(\sigma) ) ( (n, \sigma[s\rightarrow s[s_{n-1} \mapsto \sigma(s_{n-1})]] } ) when ( s_{n-1} \in \text{dom}(\sigma) )</td>
</tr>
<tr>
<td>otherwise</td>
<td>( (n - \delta + \alpha, \sigma\setminus{s_{n-1}, \ldots, s_{n-\delta}}) )</td>
</tr>
</tbody>
</table>

**Fig. 4.** Updating function

**Example 6.** Given the following initial abstract state \( \{(8, \{\}) \mapsto \{(8, \{\})\}\} \), which corresponds to the initial stack state for executing block 941, the application of the transfer function \( \tau \) to the block that starts at EVM instruction 941, produces the following results (between parenthesis we show the program point). To the right we show the application of the transfer function to block 123 with its initial abstract state \( \{(2, \{\}) \mapsto \{(2, \{\})\}\} \).

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>JUMPDEST</td>
<td>( {(8, {}) \mapsto {(8, {})}} ) ( (123) ) JUMPDEST</td>
</tr>
<tr>
<td>MD</td>
<td>( {(8, {}) \mapsto {(7, {})}} ) ( (124) ) POP</td>
</tr>
<tr>
<td>AD</td>
<td>( {(8, {}) \mapsto {(6, {})}} ) ( (125) ) PUSH2 ( 0142 )</td>
</tr>
<tr>
<td>PUS</td>
<td>( {(8, {}) \mapsto {(5, {})}} ) ( (126) ) PUSH1 ( 04 )</td>
</tr>
<tr>
<td>DUP</td>
<td>( {(8, {}) \mapsto {(4, {})}} ) ( (127) ) DUB</td>
</tr>
<tr>
<td>CALDAT</td>
<td>( {(8, {}) \mapsto {(3, {})}} ) ( (128) ) CALLDATASIZE</td>
</tr>
<tr>
<td>SWAP</td>
<td>( {(8, {}) \mapsto {(2, {})}} ) ( (129) ) SUB</td>
</tr>
<tr>
<td>STURE</td>
<td>( {(8, {}) \mapsto {(1, {})}} ) ( (130) ) CALLDATASIZE</td>
</tr>
<tr>
<td>PO</td>
<td>( {(8, {}) \mapsto {(5, {})}} ) ( (131) ) SWAP1</td>
</tr>
<tr>
<td>PUS</td>
<td>( {(8, {}) \mapsto {(7, {s_0 \mapsto 964})}} ) ( (132) ) PO</td>
</tr>
<tr>
<td>LOAD</td>
<td>( {(8, {}) \mapsto {(7, {s_0 \mapsto 964})}} ) ( (133) ) POP</td>
</tr>
<tr>
<td>PUS</td>
<td>( {(8, {}) \mapsto {(5, {s_0 \mapsto 964, s_7 \mapsto 648})}} ) ( (134) ) POP</td>
</tr>
<tr>
<td>PUS</td>
<td>( {(8, {}) \mapsto {(7, {s_0 \mapsto 964})}} ) ( (135) ) SWAP2</td>
</tr>
<tr>
<td>PO</td>
<td>( {(8, {}) \mapsto {(3, {s_1 \mapsto 142})} ) ( (136) ) POP</td>
</tr>
<tr>
<td>JUMP</td>
<td>( {(8, {}) \mapsto {(7, {s_0 \mapsto 964})}} ) ( (137) ) JUMP</td>
</tr>
<tr>
<td>SUB</td>
<td>( {(8, {}) \mapsto {(2, {})}} ) ( (138) ) JUMP</td>
</tr>
</tbody>
</table>

**Addresses equations system** The next step consists in defining, by means of the transfer and the updating functions, a constraint equation system to represent all possible jumping addresses that could be valid for executing a jump instruction in the program.

**Definition 4 (addresses equations system).** Given an EVM program \( P \) of the form \( b_0, \ldots, b_p \), its addresses equation system, \( \mathcal{E}(P) \) includes the following equations according to all EVM bytecode instruction \( b_{pc} \in P \):
consists in first creating one constraint variable \( X_{pc(s−1)} \) which is equal to \( idmap(\lambda(b_{pc}, (n, \sigma))) \) for all \( s \in \text{dom}(X_{pc}) \), \( (n, \sigma) \in X_{pc}(s) \).

(2) **JUMPI**

\[ X_{pc(s−1)} \equiv idmap(\lambda(b_{pc}, (n, \sigma))) \quad \forall s \in \text{dom}(X_{pc}), (n, \sigma) \in X_{pc}(s) \]

(3) **Jump Dest**

\[ b_{pc} \notin \text{End} \land \text{size}(b_{pc}) = \text{JUMPDEST} \]

\[ X_{pc+size(b_{pc})} \equiv idmap(\lambda(b_{pc}, (n, \sigma))) \quad \forall s \in \text{dom}(X_{pc}), (n, \sigma) \in X_{pc}(s) \]

(4) **End**

\[ b_{pc} \notin \text{End} \]

\[ X_{pc+size(b_{pc})} \equiv \tau(b_{pc}, X_{pc}) \]

(5) **otherwise**

\[ X_{pc+size(b_{pc})} \equiv \tau(b_{pc}, X_{pc}) \]

where \( idmap(s) \) returns a map \( \pi \) such that \( \text{dom}(\pi) = \{s\} \) and \( \pi(s) = \{s\} \) and \( \text{size}(b_{pc}) \) returns the number of bytes of the instruction \( b_{pc} \).

Observe that the addresses equations system will have one equation for all program points of the program. Concretely, variables of the form \( X_{pc} \) store the jumping addresses saved in the stack after executing \( b_{pc} \) for all possible entry stacks. This information will be used for computing all possible jump destinations when executing JUMP or JUMPI instructions. For computing the system, most instructions, cases (4) and (5), just apply the transfer function \( \tau \) to compute the possible stack states of the subsequent instruction. Note that the expression \( pc + \text{size}(b_{pc}) \) at (3) just computes the position of the next instruction in the EVM program. Jumping instructions, points (1) and (2), compute the initial state of the invoked blocks, thus they produce a map with all possible input stack states that can reach one block. JUMP and JUMPI instructions produce, for each stack state, one equation by taking the element from the previous stack state \( X_{\sigma(s−1)} \). JUMPI, point (2) produces an extra equation \( X_{pc+1} \) to capture the possibility of continuing to the next instruction instead of jumping to the destination address. Additionally, those instructions before \( \text{JUMPDEST} \), point (3), produce initial states for the block that starts in the \( \text{JUMPDEST} \). When the constraint equation system is solved, constraint variables over-approximate the jumping information for the program.

**Example 7.** As it can be seen if Figure 3, we can jump to block 64B from two different blocks, 941 and 123. The computation of the jump equations systems will produce the following equations for the entry program points of these two blocks:

\[
X_{941} \equiv \{(8, \{\}) \rightarrow \{(8, \{\})\}\} \\
X_{123} \equiv \{(2, \{\}) \rightarrow \{(2, \{\})\}\} \\
\vdots \\
X_{64B} \equiv \{(7, \{s_5 \rightarrow 954\}) \rightarrow \{(7, \{s_5 \rightarrow 954\})\}\} \\
X_{64A} \equiv \{(3, \{s_1 \rightarrow 142\}) \rightarrow \{(3, \{s_1 \rightarrow 142\})\}\}
\]

Observe that we have two different blocks contents reaching the same program point, e.g. two equations for \( X_{64B} \) are produced by two different blocks, the JUMP at the end of block 941, identified by \( X_{64B}^1 \), and the JUMP at the end of block 123, identified by \( X_{64B}^2 \). Thus the equation that must hold for \( X_{64B} \) is produced by the application of the operation \( X_{64B}^1 \sqcup X_{64B}^2 \), as follows:

\[ X_{64B} \equiv \{(7, \{s_5 \rightarrow 954\}) \rightarrow \{(7, \{s_5 \rightarrow 954\})\}, (3, \{s_1 \rightarrow 142\}) \rightarrow \{(3, \{s_1 \rightarrow 142\})\}\}
\]

Note that the application of the transfer function \( \tau \) for all instructions of block 64B applies function \( \lambda \) to all elements in the abstract state and updates the stack state accordingly:

\[
\begin{align*}
\text{(JUMPDEST) } X_{64B} & \equiv \{(7, \{s_5 \rightarrow 954\}) \rightarrow \{(7, \{s_5 \rightarrow 954\})\}, (3, \{s_1 \rightarrow 142\}) \rightarrow \{(3, \{s_1 \rightarrow 142\})\}\} \\
\text{(PUSH1 00) } X_{64B} & \equiv \{(7, \{s_5 \rightarrow 954\}) \rightarrow \{(8, \{s_5 \rightarrow 954\})\}, (3, \{s_1 \rightarrow 142\}) \rightarrow \{(4, \{s_1 \rightarrow 142\})\}\} \\
\text{(DUP1) } X_{64B} & \equiv \{(7, \{s_5 \rightarrow 954\}) \rightarrow \{(9, \{s_5 \rightarrow 954\})\}, (3, \{s_1 \rightarrow 142\}) \rightarrow \{(5, \{s_1 \rightarrow 142\})\}\} \\
\text{(SWAP1) } X_{64B} & \equiv \{(7, \{s_5 \rightarrow 954\}) \rightarrow \{(10, \{s_5 \rightarrow 954\})\}, (3, \{s_1 \rightarrow 142\}) \rightarrow \{(6, \{s_1 \rightarrow 142\})\}\} \\
\text{(POP) } X_{64B} & \equiv \{(7, \{s_5 \rightarrow 954\}) \rightarrow \{(9, \{s_5 \rightarrow 954\})\}, (3, \{s_1 \rightarrow 142\}) \rightarrow \{(5, \{s_1 \rightarrow 142\})\}\}
\end{align*}
\]

Solving the addresses equations system of a program \( P \) can be done iteratively. A naïve algorithm consists in first creating one constraint variable \( X_0 \equiv \pi_0([0, \sigma_0] \rightarrow \{(0, \sigma_0)\}) \), where \( \pi_0 \) and \( \sigma_0 \) are empty mappings, and \( X_{pc} \equiv \pi \bot \) for all \( pc \in P, pc \neq 0 \), and then iteratively refining the values of these variables as follows:
1. substitute the current values of the constraint variables in the right-hand side of each constraint, and then evaluate the right-hand side if needed;
2. if each constraint $X \sqsubseteq E$ holds, where $E$ is the value of the evaluation of the right-hand side of the previous step, then the process finishes; otherwise
3. for each $X \sqsubseteq E$ which does not hold, let $E'$ be the current value of $X$. Then update the current value of $X$ to $E \sqcup E'$. Once all these updates are (iteratively) applied we repeat the process at step 1.

Termination is guaranteed since the abstract domain does not have any infinitely ascending chains as the number of jump destinations and the stack size is finite.

**Example 8.** Figure 5 shows the equations produced by Definition 4 of the first and the last instruction of all blocks shown in Figure 3. The first instruction shown in the system is $X_{648}$, computed in Example 7. Observe that the application of $\tau$ stores the jumping addresses in the corresponding abstract states after PUSH instructions (see $X_{668}$, $X_{66D}$, $X_{64F}$, $X_{66C}$, . . . ). Such addresses will be used to produce the equations at the JUMP or JUMPI instructions. In the case of JUMP, as the jump is unconditional, it only produces one equation, e.g. $X_{668}$ consumes address 66F to produce the input state of $X_{668}$, or $X_{66C}$ produces the input abstract state for $X_{661}$. JUMPI instructions produce two different equations: (1) one equation which corresponds to the jumping address stored in the stack, e.g. equations $X_{66D}$ and $X_{66F}$ produced by the jumps of the equations $X_{660}$ and $X_{66D}$ respectively; and (2) one equation which corresponds to the next
instruction, e.g. $X_{661}$ and $X_{668}$ produced by $X_{660}$ and $X_{669}$, respectively. Finally, another point to highlight occurs at equation $X_{668}$: as we have two possible jumping addresses in the stack of and both can be used by the JUMP at the end of the block, we produce two inputs for the two possible jumping addresses, $X_{664}$ and $X_{142}$, for capturing the two possible branches from block 6D1 (see Figure 3).

**Theorem 1 (Soundness).** Let $P \equiv b_0, \ldots, b_p$ be a program, $X_1, \ldots, X_i$ the solution of the jumps equations system of $P$, and $pc$ the program counter of a jump instruction. Then for any execution of $P$, there exists $s \in \text{dom}(X_{pc})$ such that $(n, \sigma) \in X_{pc}(s)$ and $\sigma(s_{n-1})$ contains all jump addresses that instruction $b_{pc}$ might jump to during the execution of $P$.

**Control Flow Graph.** At this point, by means of the addresses equation system solution, we compute the control flow graph of the program. In order to simplify the notation, given a block $B_i$, we define the function $\text{getId}(i, (n, \sigma))$, which receives the block identifier $i$ and an abstract stack $(n, \sigma)$ and returns a unique identifier for the abstract stack $(n, \sigma) \in \text{dom}(X_i)$. Similarly, getJump$(i, \sigma)$ returns the jump address that corresponds to the identifier $id$ of block $B_i$. Besides, we define the function $\text{getSize}(pc, id)$ that, given a program point $pc \in B_i$ and a unique identifier $id$ for $B_i$, returns the value $n'$ s.t. $(n, \sigma) = \text{getStack}(i, id)$, and $X_{pc}(\langle n, \sigma \rangle) = \langle n', \sigma' \rangle$.

**Example 9.** Given the equation:

$$X_{648} \equiv \{ (7, \{ s_1 \rightarrow 954 \}) \rightarrow (7, \{ s_5 \rightarrow 954 \}), (3, \{ s_1 \rightarrow 142 \}) \rightarrow (3, \{ s_1 \rightarrow 142 \}) \}$$

if we compute the functions getId and getSize, we have that $\text{getId}(64B, 7, \{ s_5 \rightarrow 954 \}) = 1$ and $\text{getId}(64B, 3, \{ s_1 \rightarrow 142 \}) = 2$. Analogously, $\text{getSize}(64B, 1) = 7$ and $\text{getSize}(64B, 2) = 3$.

**Definition 5 (control flow graph).** Given an EVM program $P$, its blocks $B_i \equiv b_i \ldots b_j \in \text{blocks}(P)$ and its flow analysis results provided by a set of variables of the form $X_{pc}$ for all $pc \in P$, we define the control flow graph of $P$ as a directed graph $\text{CFG} = (V, E)$ with a set of vertices

$$V = \{ B_{i:id} | B_i \in \text{blocks}(P) \land (n, \sigma) \in \text{dom}(X_i) \land id = \text{getId}(i, (n, \sigma)) \}$$

and a set of edges $E = E_{\text{jump}} \cup E_{\text{next}}$ such that:

$$E_{\text{jump}} = \{ B_{i:id} \rightarrow B_{d:id} | b_j \in \text{Jump} \land (n, \sigma) \in \text{dom}(X_i) \land id = \text{getId}(i, (n, \sigma)) \land (n', \sigma') \in X_j((n, \sigma)) \land d = \sigma'(s_{n-1}) \land (n'', \sigma'') = \lambda(b_j, (n', \sigma')) \land id_2 = \text{getId}(d, (n'', \sigma'')) \}$$

$$E_{\text{next}} = \{ B_{i:id} \rightarrow B_{d:id} | b_j \notin \text{Jump} \land b_j \notin \text{End} \land (n, \sigma) \in \text{dom}(X_i) \land id = \text{getId}(i, (n, \sigma)) \land (n', \sigma') \in X_j((n, \sigma)) \land d = j + \text{size}(b_j) \land (n'', \sigma'') = \lambda(b_j, (n', \sigma')) \land id_2 = \text{getId}(d, (n'', \sigma'')) \}$$

The first relevant point of the control flow graph (CFG) we produce is that, for producing the set of vertices $V$, we replicate each block for each different stack state that could be used for invoking it (gray nodes in Figure 3 are replicated in the CFG). Analogously, the different entry stack states are also used to produce different edges depending on its corresponding replicated blocks. Note that the definition distinguishes between two kinds of edges. (1) Edges produced by JUMP or JUMPI instructions at the end of the blocks, whose destination is taken from the values stored in the stack states of the instruction before the jump with $d = \sigma'(s_{n-1})$; and (2) edges produced by continuations to the next instruction, whose destination is computed with $d = j + \text{size}(b_j)$. In both kinds of edges, as we could have replicated blocks, we apply function $\lambda$ and get the id of the resulting state to compute the id of the destination: $(n'', \sigma'') = \lambda(b_j, (n', \sigma')) \land id_2 = \text{getId}(d, (n'', \sigma''))$.

**Example 10.** Considering the blocks shown in Figure 3 and the equations shown at Figure 5, the CFG of the program includes non-replicated nodes for those blocks that only receive one possible stack state (white nodes in Figure 3). However, the nodes that could be reached by two different stack states (gray nodes in Figure 3) will be replicated in the CFG:

Analogously, our CFG replicates the edges according to the nodes replicated (solid and dashed edges in Figure 3):

\[ E = \{ B_{941} \rightarrow B_{648:1}, B_{648:1} \rightarrow B_{653:1}, B_{653:1} \rightarrow B_{661:1}, B_{661:1} \rightarrow B_{661:1}, B_{661:1} \rightarrow B_{66:1:1}, B_{66:1:1} \rightarrow B_{66:1:1}, \}

Note that, in Figure 3, we distinguished dashed and solid edges just to remark that we could have two possible execution paths, that is, if the call to \texttt{findWinner} comes from block \( B_{941} \), it will return to block \( B_{94} \) and, if the execution comes from a public invocation, i.e. block \( B_{123} \), it will return to block \( B_{142} \).

3 Proof for Theorem 1

The proof sketch follows the next steps:

1. We first define an \textit{EVM operational semantics} that describes how \textit{EVM} programs handle jump addresses on the stack.
2. Then we define an \textit{EVM collecting semantics} for the operational semantics. Such collecting semantics gathers all transitions that can be produced by the execution of a program \( P \).
3. We continue by defining the \textit{jumps-to property} as a property of this collecting semantics; and
4. Then we prove a lemma that states that the least solution of the set of constraints generated as described in Definition 4 is a safe approximation of the \textit{EVM} collecting semantics w.r.t. the jumps-to property.
5. Finally, we rewrite Theorem 1 in terms of the operational semantics and prove it.

Figure 6 shows the semantics of some instructions involved in the computation of the values stored in the stack for handling jumps. The state of the program \( S \) is a tuple \( \langle pc, \langle n, \sigma \rangle \rangle \) where \( pc \) is the value of the program counter with the index of the next instruction to be executed, and \( \langle n, \sigma \rangle \) is a \textit{stack state} as defined in Section 2. Interesting rules are the ones that deal with jump destination addresses on the stack: Rule (4) adds a new address on the stack, and Rules (6) and (8-10) copy or exchange existing addresses on top of the stack, respectively. Rules (1) to (3) perform a jump in the program and therefore consume the address placed on top of the stack, plus an additional word in the case of \texttt{JUMPI}. If the instructions considered in this simplified semantics do not handle jump addresses, the corresponding rules just remove some values from the stack in the program state \( S \) (Rules (5), (7) and (11)). The remaining \textit{EVM} instructions not explicitly considered in this simplified semantics are generically represented by Rule (12) with \( b_{\delta}^{\alpha} \), where \( \delta \) is the number of items removed from stack when \( b_{\delta}^{\alpha} \) is executed, and \( \alpha \) is the number of additional items placed on the stack. Complete executions are traces of the form \( S_0 \Rightarrow S_1 \Rightarrow \ldots \Rightarrow S_n \), where \( S_0 \equiv \{ 0, (0, 0) \} \) is the initial state, \( \sigma \) is the empty mapping, and \( S_n \) corresponds to the last state. There are no infinite traces, as any transaction that executes \textit{EVM} code has a finite gas limit and every instruction executed consumes some amount of gas. When the gas limit is exceeded, an out-of-gas exception occurs and the program halts immediately.

\textbf{Definition 6 (EVM collecting semantics).} Given an \textit{EVM} program \( P \), the \textit{EVM collecting semantics operator} \( \psi_p \) is defined as follows:

\[ \psi_p(X) = \{ \langle S, S' \rangle \mid \langle ., S \rangle \in X \land S \Rightarrow S' \} \]

The \textit{EVM semantics} is defined as \( \psi_p = \bigcup_{n>0} \psi_p^n(X_0) \), where \( X_0 \equiv \{ 0, (0, 0) \} \) is the initial configuration.

\textbf{Definition 7 (jumps-to property).} Let \( P \) be an IR program, \( \psi_p = \bigcup_{n>0} \psi_p^n(X_0) \), and \( b \) an instruction at program point \( pc \), then we say that \( \psi_p \vdash_{pc} T \) if \( T = \{ \langle n, \sigma \rangle \mid \langle S, S' \rangle \in \psi_p \land \langle n, \sigma \rangle \in S' \} \).

The following lemma states that the least solution of the constraint equation system defined in Definition 2 is a safe approximation of \( \psi_p \):

\textbf{Lemma 1 (soundness).} Let \( P \equiv b_0, \ldots, b_p \) be a program, \( pc \) a program point and \( X_0, \ldots, X_p \) the least solution of the constraints equation system as defined in Section 2. The following holds:

If \( \psi_p \vdash_{pc} T \), then for all \( \langle n, \sigma \rangle \in T \), exists \( s \in \text{dom}(X_{pc}) \) such that \( \langle n, \sigma \rangle \in X_{pc}(s) \).
Proof. We use $X'_{pc}$ to refer to the value obtained for $X_{pc}$ after $m$ iterations of the algorithm for solving the equation system depicted in Section 2. We say that $X_{pc}$ covers $\langle n, \sigma \rangle$ in $C^m_p(X_0)$ at program point $pc$ when this lemma holds for the result of computing $C^m_p(X_0)$. In order to prove this lemma, we can reason by induction on the value of $m$, the length of the traces $S_0 \Rightarrow^m S_m$ considered in $C^m_p(X_0)$.

Case base: if $m = 0$, $S_0 = \langle 0, (0, \sigma_0) \rangle$ and the Lemma trivially holds as $(0, \sigma_0) \in X_0^m(\langle 0, \sigma_0 \rangle)$.

Inductive Hypothesis: We assume Lemma 1 holds for all traces of length $m \geq 0$.

Inductive Case: Let us consider traces of length $m + 1$, which are of the form $S_0 \Rightarrow^m S_m \Rightarrow S_{m+1}$. $S_m$ is a program state of the form $S_m = \langle pc, (n, \sigma) \rangle$. We can apply the induction hypothesis to $S_m$; there exists some $s \in dom(X^m_{pc})$ such that $(n, \sigma) \in X^m_{pc}(s)$. For extending the Lemma, we reason for all possible rules in the simplified EVM semantics (Fig. 6) we may apply from $S_m$ to $S_{m+1}$:

- Rule (1): After executing a JUMP instruction $S_{m+1}$ is of the form $\langle \sigma(s_{n-1}), (n - 1, \sigma\backslash[s_{n-1}]) \rangle$. In iteration $m + 1$, the following set of equations corresponding to $b_{pc}$ is evaluated:

\[
X'_{\sigma(s_{n-1})} \ni \text{idmap}(\lambda(b_{pc}, \langle n', \sigma' \rangle)) \quad \text{for all } s' \in \text{dom}(X'_{pc}), (n', \sigma') \in X'_{pc}(s')
\]

where $\text{idmap}(\lambda(b_{pc}, \langle n', \sigma' \rangle)) = \pi_1(\langle n' - 1, \sigma'\backslash[s_{n-1}] \rangle \mapsto \{\langle n' - 1, \sigma'\backslash[s_{n-1}] \rangle\})$ (Case (4) in Fig. 4). The induction hypothesis guarantees that there exists some $s'' \in X^m_{pc}$ such that $(n, \sigma) \in X^m_{pc}(s'')$, and the Lemma trivially holds as $X''_{\sigma(s_{n-1})} \ni \text{idmap}(\lambda(b_{pc}, \langle n', \sigma' \rangle))$. For extending the Lemma, we reason for all possible rules in the simplified EVM semantics (Fig. 6) we may apply from $S_m$ to $S_{m+1}$:

\[
(1) \quad b_{pc} = \text{JUMP} \quad \langle pc, \langle n, \sigma \rangle \rangle \Rightarrow \langle \sigma(s_{n-1}), (n - 1, \sigma\backslash[s_{n-1}]) \rangle
\]

\[
(2) \quad b_{pc} = \text{JUMP} \quad \langle pc, \langle n, \sigma \rangle \rangle \Rightarrow \langle \sigma(s_{n-1}), (n - 2, \sigma\backslash[s_{n-1}, s_{n-2}] \rangle
\]

\[
(3) \quad b_{pc} = \text{JUMP} \quad \langle pc, \langle n, \sigma \rangle \rangle \Rightarrow \langle pc + \text{size}(b_{pc}), (n - 2, \sigma\backslash[s_{n-1}, s_{n-2}] \rangle
\]

\[
(4) \quad b_{pc} = \text{PUSH} \quad v, v \in \mathcal{J} \quad \langle pc, \langle n, \sigma \rangle \rangle \Rightarrow \langle pc + \text{size}(b_{pc}), (n + 1, \sigma[s_n \mapsto \{v\}] \rangle
\]

\[
(5) \quad b_{pc} = \text{PUSH} \quad v, v \notin \mathcal{J} \quad \langle pc, \langle n, \sigma \rangle \rangle \Rightarrow \langle pc + \text{size}(b_{pc}), (n + 1, \sigma) \rangle
\]

\[
(6) \quad b_{pc} = \text{DUP} \quad s_{n-x} \in \text{dom}(\sigma) \quad \langle pc, \langle n, \sigma \rangle \rangle \Rightarrow \langle pc + \text{size}(b_{pc}), (n + 1, \sigma[s_n \mapsto \sigma(s_{n-x})]) \rangle
\]

\[
(7) \quad b_{pc} = \text{DUP} \quad s_{n-x} \notin \text{dom}(\sigma) \quad \langle pc, \langle n, \sigma \rangle \rangle \Rightarrow \langle pc + \text{size}(b_{pc}), (n + 1, \sigma) \rangle
\]

\[
(8) \quad \langle pc, \langle n, \sigma \rangle \rangle \Rightarrow \langle pc + \text{size}(b_{pc}), (n, \sigma[s_{n-x-1} \mapsto \sigma(s_{n-1}), s_{n-1} \mapsto \sigma(s_{n-1})]) \rangle
\]

\[
(9) \quad n \notin \text{dom}(\sigma) \quad \langle pc, \langle n, \sigma \rangle \rangle \Rightarrow \langle pc + \text{size}(b_{pc}), (n, \sigma[s_{n-x-1} \mapsto \sigma(s_{n-1})\backslash[s_{n-1}]) \rangle
\]

\[
(10) \quad n \notin \text{dom}(\sigma) \quad \langle pc, \langle n, \sigma \rangle \rangle \Rightarrow \langle pc + \text{size}(b_{pc}), (n, \sigma[s_{n-x-1} \mapsto \sigma(s_{n-1})\backslash[s_{n-1}]) \rangle
\]

\[
(11) \quad n \notin \text{dom}(\sigma) \quad \langle pc, \langle n, \sigma \rangle \rangle \Rightarrow \langle pc + \text{size}(b_{pc}), (n, \sigma[s_{n-1}, s_{n-x-1}] \rangle
\]

\[
(12) \quad b_{pc} \in \text{otherwise}, b_{pc} \notin \text{End} \quad \langle pc, \langle n, \sigma \rangle \rangle \Rightarrow \langle pc + \text{size}(b_{pc}), (n - \delta + \alpha, \sigma[s_{n-1}, \ldots, s_{n-\delta}] \rangle
\]

Fig. 6. Simplified EVM semantics for handling jumps
where $S_m = \langle pc, \langle n, \sigma \rangle \rangle$. Therefore, at iteration $m + 1$, the following must hold:

$$X_{\sigma(n-1)}^{m+1} \sqsubseteq \pi_\bot [(n-1, \sigma \setminus [s_{n-1}]) \mapsto \{ (n-1, \sigma \setminus [s_{n-1}]) \}]$$

so $\langle n-1, \sigma \setminus [s_{n-1}] \rangle \in X_{\sigma(n-1)}^{m+1} (\langle n-1, \sigma \setminus [s_{n-1}] \rangle)$ and thus Lemma 1 holds.

- Rules (2) and (3): After executing a JUMP instruction, $S_{m+1}$ is either $\langle s(n-1), \langle n-2, \sigma \setminus [s_{n-1}, s_{n-2}] \rangle \rangle$ or $\langle pc + \text{size}(b_{pc}), \langle n-2, \sigma \setminus [s_{n-1}, s_{n-2}] \rangle \rangle$, respectively. In any of those cases the following sets of equations are evaluated:

$$X_{\sigma(s_{n-2})} \equiv \text{idmap}(\lambda(\text{JUMP}, \langle n', \sigma' \rangle)) \quad \text{for all } s' \in \text{dom}(X_{pc}(n'))$$

$$X_{\sigma(s_{n-2})}^{m+1} \sqsubseteq \pi_\bot [(n-2, \sigma \setminus [s_{n-1}, s_{n-2}]) \mapsto \{ (n-2, \sigma \setminus [s_{n-1}, s_{n-2}]) \}]$$

where idmap$(\lambda(b_{pc}, \langle n', \sigma' \rangle)) = \pi_\bot [(n' - 2, \sigma' \setminus [s_{n-1}, s_{n-2}]) \mapsto \{ (n' - 2, \sigma' \setminus [s_{n-1}, s_{n-2}]) \}]$ (Case 4) of the definition of the update function $\lambda$ in Fig. 4). As in the previous case, the induction hypothesis guarantees that at Iteration $m$ there exists $s'' \in X_{pc}^m$ such that $\langle n, \sigma \rangle \in X_{pc}^m(s'')$. Therefore, in Iteration $m + 1$, the following must hold:

$$X_{\sigma(s_{n-2})}^{m+1} \sqsubseteq \pi_\bot [(n-2, \sigma \setminus [s_{n-1}, s_{n-2}]) \mapsto \{ (n-2, \sigma \setminus [s_{n-1}, s_{n-2}]) \}]$$

and thus Lemma 1 holds for these cases as well.

- Rules (4) - (12): We will first consider the case in which any of these rules corresponds to an EVM instruction followed by an instruction different from JUMPDEST. All rules are similar, as they all use the set of equations generated by Case (4) in Definition 4. We will see Rule (4) in detail. After executing a PUSH $v$ instruction, $S_{m+1}$ is $\langle pc + \text{size}(b_{pc}), \langle n+1, \sigma(s_{n} \mapsto \{ v \}) \rangle \rangle$. We have to prove that there exists some $s \in \text{dom}(X_{pc+\text{size}(b_{pc})}(s))$ such that $\langle n+1, \sigma(s_{n} \mapsto \{ v \}) \rangle \in X_{pc+\text{size}(b_{pc})}(s)$. The following set of equations is evaluated:

$$X_{pc+\text{size}(b_{pc})} \sqsubseteq \tau(\text{PUSH}, X_{pc}) \quad (1)$$

By Definition 3, $\tau(\text{PUSH}, X_{pc}) = \pi'$, where $\forall s' \in \text{dom}(\pi)$, $\pi(s') = \lambda(\text{PUSH}, X_{pc}(s'))$. By the case (1) of the definition of the update function $\lambda$, we have that:

$$\forall (n'', \sigma'') \in \text{dom}(X_{pc}(n'')) \land \pi'((n'', \sigma'')) = \langle n'' + 1, \sigma'' \mapsto \{ v \} \rangle \quad (2)$$

By the induction hypothesis, at Iteration $m$ there exists some $s \in \text{dom}(X_{pc}^m(s))$ such that $\langle n, \sigma \rangle \in X_{pc}^m(s)$. Therefore, by 1 and 2, at Iteration $m + 1$ we have that the following holds:

$$s \in \text{dom}(X_{pc+\text{size}(b_{pc})}(s')) \quad (3)$$

and thus Lemma 1 holds for Rule (4).

- Rules (4) - (12), followed by a JUMPDEST instruction. After executing any of these instructions, $S_{m+1}$ is $\langle pc + \text{size}(b_{pc}), \langle n''', \sigma''' \rangle \rangle$, where $\langle n''', \sigma''' \rangle$ is obtained according to the rule from Figure 6. We have to prove that there exists some $s \in \text{dom}(X_{pc+\text{size}(b_{pc})}(s))$ such that $\langle n'', \sigma'' \rangle \in X_{pc+\text{size}(b_{pc})}(s)$. The following set of equations is evaluated:

$$X_{pc+\text{size}(b_{pc})} \sqsubseteq \text{idmap}(\lambda(b_{pc}, \langle n', \sigma' \rangle)) \quad \text{for all } s' \in \text{dom}(X_{pc}(n')), \langle n', \sigma' \rangle \in X_{pc}(s')$$

where idmap$(\lambda(b_{pc}, \langle n', \sigma' \rangle)) = \pi_\bot [(n', \sigma') \mapsto \{ (n', \sigma') \}]$ (where $n'$ and $\sigma'$ are obtained according to the cases of the updating function detailed in Figure 4. We can see that $\{ (n', \sigma') \}$ match the modification made to the state $S_{m+1}$ by the corresponding rule of the semantics. Therefore, at Iteration there exists an $s = \{ (n', \sigma') \}$ such that $\{ (n', \sigma') \} \in X_{pc+\text{size}(b_{pc})}^{m+1}$ and Lemma 1 also holds.

When the algorithm stops Lemma 1 holds, as for any pc $X_{pc}^{m+1} \sqsubseteq X_{pc}$ for each iteration of the algorithm for solving the equations system of Section 2.

Now we rewrite Theorem 1 in terms of the operational semantics of Figure 6. This rewriting actually is stronger than Theorem 1, as guarantees the correctness of the stack states obtained from the jumps equations system at any step of the execution.

**Theorem 2 (Soundness).** Let $P \equiv b_0, \ldots, b_n$ be a program, $S_0 = \langle 0, (0, \sigma) \rangle$ the initial program state, and $X_1, \ldots, X_n$ the solution of the jumps equations system of $P$. Then for any trace $S_0 \leadsto^* S_m$, where $S_m = \langle pc, \langle n, \sigma \rangle \rangle$, there exists $s \in \text{dom}(X_{pc})$ such that $\langle n, \sigma \rangle \in X_{pc}(s)$.

**Proof.** Straightforward from Lemma 1, as the EVM collecting semantics takes into account all possible traces of the operational semantics.
References


