Formally Verified EVM Block-Optimizations* (Extended Version)**

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Abstract. The efficiency and the security of *smart contracts* are their two fundamental properties, but might come at odds: the use of optimizers to enhance efficiency may introduce bugs and compromise security. Our focus is on EVM (Ethereum Virtual Machine) *block-optimizations*, which enhance the efficiency of jump-free blocks of opcodes by eliminating, reordering and even changing the original opcodes. We reconcile efficiency and security by providing the verification technology to formally prove the correctness of EVM block-optimizations on smart contracts using the Coq proof assistant. This amounts to the challenging problem of proving semantic equivalence of two blocks of EVM instructions, which is realized by means of three novel Coq components: a symbolic execution engine which can execute an EVM block and produce a symbolic state; a number of simplification lemmas which transform a symbolic state into an equivalent one; and a checker of symbolic states to compare the symbolic states produced for the two EVM blocks under comparison.

Keywords: Coq \cdot Ethereum Virtual Machine \cdot Smart Contracts \cdot Optimization \cdot Theorem Proving.

1 Introduction

In many contexts, security requirements are critical and formal verification today plays an essential role to verify/certify these requirements. One of such contexts is the blockchain, in which software bugs on *smart contracts* have already caused several high profile attacks (e.g., [16,15,28,35,14,13]). There is hence huge interest and investment in guaranteeing their correctness, e.g., Certora [1], Veridise

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[2], apriorit [3], Consensys [4], Dedaub [5] are companies that offer smart contract audits using formal methods' technology. In this context, efficiency is of high relevance as well, as deploying and executing smart contracts has a cost (in the corresponding cryptocurrency). Hence, optimization tools for smart contracts have emerged in the last few years (e.g., ebso [27], SYRUP [11], GASOL [10], the solc optimizer [9]). Unfortunately, there is a dichotomy of efficiency and correctness: as optimizers can be rather complex tools (not formally verified), they might introduce bugs and potential users might be reluctant of optimizing their code. This has a number of disruptive consequences: owners will pay more to deploy (non-optimized) smart contracts; clients will pay more to run transactions every time they are executed; the blockchain will accept less transactions as they are more costly. Rather than accepting such a dichotomy, our work tries to overturn it by developing a fully automated formal verification tool for proving the correctness of the optimized code.

The general problem addressed by the paper is formally verifying semantic equivalence of two bytecode programs, an initial code I and an optimization of it **O** –what is considered a great challenge in formal verification. For our purpose, we will narrow down the problem by (1) considering fragments of code that are *jump-free* (i.e., they do not have loops nor branching), and by (2) considering only stack EVM operations (memory/storage opcodes and other blockchainspecific opcodes are not considered). These assumptions are realistic as working on jump-free blocks still allows proving correctness for optimizers that work at the level of the blocks of the CFG (e.g., super-optimizers [27,11,10] and many rule-based optimizations performed by the Solidity compiler [9]). Considering only stack optimizations, and leaving out memory and storage simplifications and blockchain-specific bytecodes, does not restrict the considered programs, as we work at the smaller block partitions induced by the not handled operations found in the block (splitting into the block before and after). Even in our narrowed setting, the problem is challenging as block-optimizations can include any elimination, reorder and even change of the original bytecodes.

Consider the next block I, taken from a real smart contract [8]. The GASOL optimizer [10], relying on the commutativity of OR and AND, optimizes it to O:

I: PUSH2 0x100 PUSH1 0x1 PUSH1 0xa8 SHL SUB NOT SWAP1 SWAP2 AND PUSH1 0x8 SWAP2 SWAP1

- SWAP2 SHL PUSH2 0x100 PUSH1 0x1 PUSH1 0xa8 SHL SUB AND OR PUSH1 0x5
- O: PUSH2 0x100 PUSH1 0x1 PUSH1 0xa8 SHL SUB DUP1 NOT SWAP2 PUSH1 0x8 SHL AND SWAP2 AND OR PUSH1 0x5

This saves 11 bytes because (1) the expression SUB(SHL(168,1),256) —that corresponds to "PUSH2 0x100 PUSH1 0x1 PUSH1 0xa8 SHL SUB" — is computed twice; but it can be duplicated if the stack operations are properly made saving 8 bytes; and (2) two SWAPs are needed instead of 5, saving 3 more bytes.

This paper proposes a technique, and a corresponding tool, to automatically verify the correctness of EVM block-optimizations (as those above) on smart contracts using the Coq proof assistant. This amounts to the challenging problem of proving semantic equivalence of two blocks of EVM instructions, which is realized by means of three main components which constitute our main contributions (all formalized and proven correct in Coq): (1) a symbolic interpreter in Coq to symbolically execute the EVM blocks I and O and produce resulting symbolic states S_{I} and S_{0} , (2) a series of simplification rules, which transform S_{I} and S_{0} into equivalent ones S'_{I} and S'_{0} , (3) a checker of symbolic states in Coq to decide if two symbolic states S'_{I} and S'_{0} are semantically equivalent.

2 Background

The Ethereum VM (EVM) [36] is a stack-based VM with a word size of 256-bits that is used to run the smart contracts on the Ethereum blockchain. The EVM has the following categories of bytecodes: (1) Stack operations; (2) Arithmetic operations; (3) Comparison and bitwise logic operations; (4) Memory and storage manipulation; (5) Control flow operations; (6) Blockchain-specific opcodes, e.g., block and transaction environment information, compute hash, calls, etc. The first three types of opcodes are handled within our verifier, and handling optimizations on opcodes of types 4-6 is discussed in Sec. 6.

The focus of our work is on optimizers that perform optimizations only at the level of the blocks of the CFG (i.e., intra-block optimizations). A well-known example is the technique called *super-optimization* [24] which, given a loop-free sequence of instructions searches for the optimal sequence of instructions that is semantically equivalent to the original one and has optimal cost (for the considered criteria). This technique dates back to 1987 and has had a revival [29,23] thanks to the availability of SMT solvers that are able to do the search efficiently. We distinguish two types of possible intra-block optimizations: (*i*) Rule-based optimizations which consist in applying arithmetic/bitwise simplifications like ADD(X, O)=X or NOT(NOT(X))=X (see a complete list of these rules in App. A); and (*ii*) Stack-data optimizations which consist in searching for alternative stack operations that lead to an output stack with exactly the same data.

Example 1 (Intra-block optimizations). The rule-based optimization (i) $X+0\rightarrow X$ simplifies the block "PUSH1 0x5, PUSH1 0x0, ADD" to "PUSH1 0x5". On the other hand, stack-data optimizations (ii) can optimize to "ADD, DUP1" the block "DUP2, DUP2, ADD, SWAP2, ADD", as duplicating the operands and repeating the ADD operation is the same as duplicating the result. Unlike rule-based optimization, stack-data optimizations cannot be expressed as simple patterns that can be easily recognized.

The first type of optimizations are applied by the optimizer integrated in the Solidity compiler [9] as rule transformations, and they are also applied by EVM optimizers in different ways. ebso [27] encodes the semantics of arithmetic and bitwise operations in the SMT encoding so that the SMT solver searches for these optimizations together with those of type (ii). Instead, SYRUP [11] and GASOL [10] apply rule-based optimizations in a pre-phase and leave to the SMT solver only the search for the second type of optimizations. This classification of optimizations is also relevant for our approach as (i) will require integrating and proving all simplification rules correct (Sec. 4.2) while (ii) are implicit within the symbolic execution (Sec. 4.1). A block of EVM code that has been subject to optimizations of the two types above is in principle "provable" using our tool.

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There is not much work yet on formalizing the EVM semantics in Coq. One of the most developed approaches is [20], which is a definition of the EVM semantics in the Lem [26] language that can be exported to interactive theorem provers like Isabelle/HOL or Coq. According to the comparison in [19], this implementation of EVM "is executable and passes all of the VM tests except for those dealing with more complicated intercontract execution". However, we have decided not to use it for our checker due to three reasons: (a) the generated Coq code from Lem definitions is not "necessarily idiomatic" and thus it would generate a very complex EVM formalization in Coq that would make theorems harder to state and prove; (b) the author of the Lem definition states that "the Coq version of the EVM definition is highly experimental"; and (c) it is not kept up-to-date.

The other most developed implementation of the EVM semantics in Coq that we have found is [21]. It supports all the basic EVM bytecodes we consider in our checker, and looked promising as our departing point. The implementation uses *Bedrock Bit Vectors (bbv)* [7] for representing the EVM 256-bit values, as we use as well. It is not a full formalization of the EVM because it does not support calling or creation of smart contracts, but provides a function that simulates consequent application of opcodes to the given execution state, call info and Ethereum state mocks. The latter two pieces of information would add complexity and are not needed for our purpose. Therefore, we decided to develop our own EVM formalization in Coq (presented in Sec. 3) which builds upon some ideas of [21], but introduces only the minimal elements we need to handle the instructions supported by the checker. This way the proofs will be simpler and conciser.

3 EVM Semantics in Coq

Our EVM formalization is a concrete interpreter that executes a block of EVM instructions. For representing EVM words we use EVMWord that stands for the type "word 256" of the bbv library [7]. For representing instructions we use:

Inductive stack_op_instr :=	Inductive instr:=
ADD	PUSH (size: nat) (w: EVMWord)
MUL	POP
SUB	DUP (pos: nat)
DIV	SWAP (pos: nat)
NOT.	StackInstr (label: stack_op_instr).

Type stack_oper_instr defines instructions that operate only on the stack, i.e., each pops a fixed number of elements and pushes a single value back (see App. B for the full list). Type instr encapsulates this category together with the stack manipulation instruction (PUSH, etc). The type block stands for "list instr".

To keep the framework general, and simplify the proofs, the actual implementation of instructions from stack_op_instr are provided to the interpreter as input. For this, we use a map that associates instructions to implementations:

The type stack_operation defines an implementation for a given operation: comm indicates if the operation is commutative; n is the number of stack elements to be removed and passed to the operation; and f is the actual implementation. The type stack_op_map maps keys of type stack_oper_instr to values of type stack_operation. Suppose evm_add and evm_mul are implementations of ADD and MUL (see App. C), the actual stack operations map is constructed as follows:

Definition evm_stack_opm : stack_op_map :=	
$\texttt{ADD} \mid \rightarrow \texttt{i StackOp true } 2 \texttt{ evm_add} \texttt{; MUL} \mid \rightarrow \texttt{i StackOp true } 2 \texttt{ evm_mul} \texttt{; } \dots$	

In addition, we require the operations in the map to be valid with respect to the properties that they claim to satisfy (e.g., commutativity), and that when applied to the right number of arguments they should succeed (i.e., do not return None). We refer to this property as valid_stack_op_map.

An execution state (or simply state) includes only a stack (currently we support only stack operations) which is as a list of EVMWord, and the interpreter is a function that takes a block, an initial state, and a stack operations map, and iteratively executes each of the block's instructions:

```
Definition stack := list EVMWord.
Inductive state :=
  | ExState (stk: stack).
Fixpoint concr_int (p: block) (st: state) (ops: stack_op_map): option state := ...
```

The result can be either Some st or None in case of an error which are caused only due to stack overflow. In particular, we are currently not taking into account the amount of gas needed to execute the block. Our implementation follows the EVM semantics [36], considering the simplicity of the supported operations, the concrete interpreter is a minimal trusted computing base. In the future, we plan to test it using the EVM test suite.

4 Formal Verification of EVM-Optimizations in Coq

Two jump-free blocks p1 and p2 are equivalent wrt. to an initial stack size k, if for any initial stack of size k, the executions of p1 and p2 succeed and lead to the same state. Formally:

```
Definition sem_eq_blocks: (p1 p2: block) (k: nat) (ops: stack_op_map) : Prop :=
 ∀ (in_st: state) (in_stk: stack),
 get_stack in_st = in_stk → length in_stk = k →
 ∃ (out_st : state), concr_int p1 in_st ops = Some out_st ∧
 concr_int p2 in_st ops = Some out_st
```

Note that when concr_int returns None for both p1 and p2, they are not considered equivalent because in the general case they can fail due to different reasons. Note also that EVM operations are deterministic, so if concr_int evaluates to a successful final state out_st it will be unique.

An EVM block equivalence checker is a function that takes two blocks, the size of the initial stack, and returns true/false. Providing the size k of the initial stack is not a limitation of the checker, as this information is statically known in advance. Note that the maximum stack size in EVM is bounded by 1024, and

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that if the execution (of one or both blocks) wrt. to this concrete initial stack size leads to under/over stack overflow they cannot be reported equivalent. The soundness of the equivalence checker is stated as follows:

efinition	eq_block_chkr_snd	(chkr : 1	t block o bl	Lock $ ightarrow$ nat	$\mathtt{t} ightarrow \mathtt{bool}$) : Pr op :=
\forall (p1 p2:	<pre>block) (k: nat),</pre>					
chkr	$p1 p2 k = true \rightarrow set$	m equiv	blocks p1	p2 k evm	stack or	m

Given two blocks p_1 and p_2 , checking their equivalence (in Coq) has the following components: (i) <u>Symbolic Execution (Sec. 4.1)</u>: it is based on an interpreter that symbolically executes a block, wrt. an initial symbolic stack of size k, and generates a final symbolic stack. It is applied on both p_1 and p_2 to generate their corresponding symbolic output states S_1 and S_2 . (ii) <u>Rule optimizations</u> (Sec. 4.2): it is based on simplification rules that are often applied by program optimizers, which rewrite symbolic states to equivalent "simpler" ones. This step simplifies S_1 and S_2 to S'_1 and S'_2 . (iii) <u>Equivalence Checker (Sec. 4.3)</u>: it receives the simplified symbolic states, and determines if they are equivalent for any concrete instantiation of the symbolic input stack. It takes into account, for example, the fact that some stack operations are commutative.

4.1 EVM Symbolic Execution in Coq

Symbolic execution takes an initial symbolic state (i.e., stack) $[s_0, \ldots, s_k]$, a block, and a map of stack operations, and generates a final symbolic state (i.e., stack) with symbolic expressions, e.g., $[5+s_0, s_1, s_2]$, representing the corresponding computations. In order to incorporate rule-based optimizations in a simple and efficient way, we want to avoid compound expressions such as $5 + (s_0 * s_1)$, and instead use temporal fresh variables together with a corresponding map that assigns them to simpler expressions. E.g., the stack $[5 + (s_0 * s_1), s_2]$ would be represented as a tuple $([e_1, s_2], \{e_1 \mapsto 5 + e_0, e_0 \mapsto s_0 * s_1\})$ where e_i are fresh variables. To achieve this, we define the symbolic stack as a list of elements that can be numeric constant values, initial stack variables or fresh variables:

```
Inductive sstack_val : Type :=
    | Val (val: EVMWord) | InStackVar (var: nat) | FreshVar (var: nat).
Definition sstack := list sstack_val.
```

and the map that assigns meaning to fresh variables is a list that maps each fresh variable to a sstack_val, or to a compound expression:

```
Inductive smap_val : Type :=
    | SymBasicVal (val: sstack_val)
    | SymOp (opcode : stack_op_instr) (args : list sstack_val).
Definition smap := list (nat*smap_val).
```

Finally, a symbolic state is defined as a SymState term where k is the size of the initial stack, maxid is the maximum id used for fresh variables (kept for efficiency), sstk is a symbolic stack, and m is the map of fresh variables.

Inductive sstate: Type := | SymState (k maxid: nat) (sstk: sstack) (m: smap).

Example 2 (Symbolic execution). Given $p_1 \equiv$ "PUSH1 0x5 SWAP2 MUL ADD" and $p_2 \equiv$ "PUSH1 0x0 ADD MUL PUSH1 0x5 ADD", symbolically executing them with

k=3 we obtain the symbolic states represented by **sst1** ≡ ($[e'_1, s_2], \{e'_1 \mapsto e'_0 + 5, e'_0 \mapsto s_1 * s_0\}$) and **sst2** ≡ ($[e_2, s_2], \{e_2 \mapsto 5 + e_1, e_1 \mapsto e_0 * s_1, e_0 \mapsto 0 + s_0\}$).

Note that we impose some requirements on symbolic states to be valid. E.g., for any element $i \mapsto v$ of the fresh variables map, all fresh variables that appear in v have smaller indices than i. We refer to these requirements as valid_sstate.

Given a symbolic (final) state and a concrete initial state, we can convert the symbolic state into a concrete one by replacing each s_i by its corresponding value, and evaluating the corresponding expressions (following their definition in the stack operations map). We have a function to perform this evaluation that takes the stack operations map as input:

```
Definition eval_sstate (in_st: state) (sst: sstate) (ops : stack_op_map)
  : option state := ...
```

Our symbolic execution engine is a function that takes the size of the initial stack, a block, a map of stack operations, and generates a symbolic final state:

 $extsf{Definition sym_exec (p: block) (k: nat) (ops: stack_op_map) : option sstate := ...}$

Note that we do not pass an initial symbolic state, but rather we construct it inside using k. Also, the result can be None in case of failure (the causes are the same as those of conc_interpreter).

Soundness of sym_exec means that whenever it generates a symbolic state as a result, then the concrete execution from any stack of size k will succeed and produce a final state that agrees with the generated symbolic state:

```
Theorem sym_exec_snd:

∀ (p: block) (k: nat) (ops: stack_op_map) (sst: sstate),

valid_stack_op_map ops →

sym_exec p k ops = Some sst →

valid_sstate sst ∧

∀ (in_st: state) (in_stk : stack),

get_stack in_st = in_stk →

length in_stk = k →

∃ (out_st : state),

concr_int p in_st ops = Some out_st ∧

eval_sstate in_st sst ops = Some out_st
```

4.2 Simplification Rules

To capture equivalence of programs that have been optimized according to "rule simplifications" (type (i) in Sec. 2) we need to include the same type of simplifications (see App. A) in our framework. Without this, we will capture EVM-blocks equivalence only for "data-stack equivalence optimizations" (type (ii) in Sec. 2).

An optimization function takes as input a symbolic state, and tries to simplify it to an equivalent state. E.g., if a symbolic state includes $e_i \mapsto s_3 + 0$, we can replace it by $e_i \mapsto s_3$. The following is the type used for optimization functions:

 $extsf{Definition optim} := extsf{sstate} o extsf{sstate*bool}.$

Optimization functions never fail, i.e., in the worst case they return the same symbolic state. This is why the returned value includes a Boolean to indicate if any optimization has been applied, which is useful when composing optimizations later. The soundness of an optimization function can be stated as follows:

So far we have implemented and proven correct the most-used simplification rules (see App. A). E.g., there is an optimization function $optimize_add_zero$ that rewrites expressions of the form E + 0 or 0 + E to E, and its soundness theorem is:

Theorem optimize_add_zero_snd: optim_snd optimize_add_zero.

Example 3. Consider again the blocks of Ex. 2. Using optimize_add_zero we can rewrite sst2 to sst2' \equiv ([e_2, s_2], { $e_2 \mapsto 5 + e_1, e_1 \mapsto e_0 * s_1, e_0 \mapsto s_0$ }), by replacing $e_0 \mapsto 0 + s_0$ by $e_0 \mapsto s_0$.

Note that the checker can be easily extended with new optimization functions, simply by providing a corresponding implementation and a soundness proof. Optimization functions can be combined to define *simplification strategies*, which are also functions of type optim. E.g., assuming that we have *basic* optimization functions $f_1,...,f_n$: (1) Apply $f_1,...,f_n$ iteratively such that in iteration *i* function f_i is applied as many times as it can be applied. (2) Apply each f_i once in some order and repeat the process as many times as it can be applied. (3) Use the simplifications that were used by the optimizer (it needs to pass these hints).

4.3 Stacks Equivalence Modulo Commutativity

We say that two symbolic stacks sst1 and sst2 are equivalent if for every possible initial concrete state st they evaluate to the same state. Formally:

However, this notion of semantic equivalence is not computable in general, and thus we provide an effective procedure to determine such equivalence by checking that at every position of the stack both contain "similar" expressions:

Definition eq_sstate_chkr (sst1 sst2: sstate) (ops : stack_op_map) : bool := ...

To determine if two stack elements are similar, we follow their definition in the map if needed until we obtain a value that is not a fresh variable, and then either (1) both are equal constant values; (2) both are equal initial stack variables; or (3) both correspond to the same instruction and their arguments are (recursively) equivalent (taking into account the commutativity of operations). E.g., the stack elements (viewed as terms) $\text{DIV}(\text{MUL}(s_0,\text{ADD}(s_1,s_2)),0x16)$ and $\text{DIV}(\text{MUL}(\text{ADD}(s_2,s1),s_0),0x16)$ are considered equivalent because the operations ADD and MUL are commutative.

Example 4. eq_sstate_chkr fails to prove equivalence of sst1 and sst2 of Ex. 2, because, when comparing e_2 and e'_1 , it will eventually check if $0 + s_0$ and s_0 are equivalent. It fails because the comparison is rather "syntactic". However, it succeeds when comparing sst1 and sst2' (Ex. 3), which is a simplification of sst2.

This procedure is an approximation of the semantic equivalence, and it can produce false negatives if two symbolic states are equivalent but are expressed with different syntactic constructions. However, it is sound:

Note that we require the stack operations map to be valid in order to guarantee that the operations declared commutative in ops are indeed commutative. In order to reduce the number of false negatives, the simplification rules presented in Sec. 4.2 are very important to rewrite symbolic states into closer syntactic shapes that can be detected by eq_sstate_chkr.

Finally, given all the pieces developed above, we can now define the block equivalence checker as follows:

```
Definition evm_eq_block_chkr (opt: optim) (p1 p2: block) (k: nat) : bool :=
match sym_exec p1 k evm_stack_opm with
| None ⇒ false
| Some sst1 ⇒
match sym_exec p2 k evm_stack_opm with
| None ⇒ false
| Some sst2 ⇒ let (sst2', _) := opt sst1 in
let (sst1', _) := opt sst2 in
eq_sstate_chkr sst1' sst2' evm_stack_opm
end
and
```

```
end.
```

It symbolically executes p1 and p2, simplifies the resulting symbolic states by applying optimization opt, and finally calls eq_sstate_chkr to check if the states are equivalent. Note that it is important to apply the optimization rules to both blocks, as the checker might apply optimization rules that were not applied by the external optimizer. This would lead to equivalent symbolic states with different shapes that will not be detected by the symbolic state equivalence checker.

The above checker is sound when opt is sound:

```
Theorem evm_eq_block_chkr_snd: \forall (opt: optim), optim_snd opt \rightarrow eq_block_chkr_snd (evm_eq_block_chkr opt)
```

5 Implementation and Experimental Evaluation

The different components of the tool have been implemented in Coq v8.15.2, together with complete proofs of all the theoretical results (more than 180 proofs in \sim 7000 lines of Coq code). The code can be found at https://github.com

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	#blocks	CH	KR		\mathbf{KR}^{s}				#blocks	CH	IKR		$\mathbf{K}\mathbf{R}^{s}$
SIMP		Yes	Time	Yes	Time		S	IMP		Yes	Time	Yes	Time
$\overline{\text{GAS}}$ \checkmark \times	$\left \begin{array}{c} 36624\\ 43228 \end{array}\right $	$\begin{vmatrix} 36624 \\ 27149 \end{vmatrix}$	$\begin{array}{c} 2.60 \\ 4.69 \end{array}$	$\begin{vmatrix} 36624 \\ 43109 \end{vmatrix}$	$\begin{array}{c} 11.76\\ 14.09 \end{array}$	OT 7 F	SIZE	× √	$35754 \\ 32192$	$\ 3575\ 3148$	$\begin{array}{c} 4 & 2.57 \\ 8 & 2.50 \end{array}$	$ 35754 \\ 31798$	$\begin{array}{c} 12.59 \\ 12.17 \end{array}$

Table 5.1. Summary of experiments using GASOL.

/costa-group/forves/tree/stack-only. The tool currently includes 15 simplification rules (see App. A). We have tried our implementation on the outcome of two optimization tools: (1) the standalone GASOL optimizer and, (2) the optimizer integrated within the official Solidity compiler solc. For (1), we have already fully automated the communication among the optimizer and checker and have been able to perform a thorough experimental evaluation. While in (2), the communication is more difficult to automate because the CFG of the original program can change after optimization, i.e., it can make crossblock optimization. Hence, in this case, we have needed human intervention to disable intra-block optimizations and obtain the blocks for the comparison (we plan to automate this usage in the future). For evaluating (2) we have used as benchmarks 1,280 blocks extracted from the smart contracts in the semantic test suite of the solc compiler [6], succeeding to prove equivalence on 1,045 out of them. We have checked that the fails are due to the use of optimization rules not yet implemented by us. As these blocks are obtained from the test suite of the official **solc** Solidity compiler, optimized using the **solc** optimizer, the good results on this set suggest the validity can be generalized to other optimizers. Now we describe in detail the experimental evaluation on (1) for which we have used as benchmarks 147,798 blocks belonging to 96 smart contracts (see App. D).

GASOL allows enabling/disabling the application of simplification rules and choosing an optimization criteria: GAS consumption or bytes SIZE (of the code) [10]; combining these parameters we obtain 4 different sets of pairs-ofblocks to be verified by our tool. From these blocks, we consider only those that were actually optimized by GASOL, i.e., the optimized version is syntactically different from the original one. In all the cases, the average size of blocks is 8 instructions. Table 5.1 summarizes our results, where each row corresponds to one setting out of the 4 mentioned above: Column 1 includes the optimization criteria; Column 2 indicates if rule simplifications were applied by GASOL; Column 3 indicates how many pairs-of-blocks were checked; Columns 4-7 report the results of applying 2 versions of the checker, namely CHKR corresponds to the checker that only compares symbolic states and $CHKR^{s}$ corresponds to the checker that also applies all the implemented rule optimizations iteratively as much as they can be applied (see Sect. 4.2). For each we report the number of instances it proved equivalent and the total runtime in seconds. The experiments have been performed on a machine with an Intel i7-4790 at 3.60 GHz and 16GB of RAM.

For sets in which GASOL does not apply simplification rules (marked with \times), both CHKR and CHKR^s succeed to prove equivalence of all blocks. When simplifications are applied (marked with \checkmark), CHKR^s succeeds in 99% of the blocks while CHKR ranges from 63% for GAS to 99% for SIZE. This difference is due to the fact that GASOL requires the application of rules to optimize more blocks wrt. GAS (~ 37% of the total) than wrt. SIZE (~ 1%). Moreover, all the blocks that CHKR^s cannot prove equivalent have been optimized by GASOL using rules which are not currently implemented in the checker, so we predict a success rate of 100% when all the rules in App. A are integrated. Regarding time, CHKR^s is 3–5 times slower than CHKR because of the overhead of applying rule optimizations, but it is still very efficient (all 147.798 instances are checked in 50.61 seconds). As a final comment, thanks to the checker we found a bug in the parsing component of GASOL, that treated the SGT bytecode as GT. The bug was directly reported to the GASOL developers and is already fixed.⁴

6 Conclusions, Related and Future Work

Our work provides the first tool able to formally verify the equivalence of jumpfree EVM blocks and has required the development of all components within the verification framework. The implementation is not tied to any specific tool and could be easily integrated within any optimization tool. Ongoing work focuses on handling memory and storage optimizations. This extension needs to support the execution of memory/storage operations at the level of the concrete interpreter, and design an efficient data structure to represent symbolic memory/storage. Full handling of blockchain-specific opcodes is straightforward, it only requires adding the corrsponding implementations to the stack operations map evm_stack_opm. A more ambitious direction for future work is to handle cross-block optimizations.

There are two approaches to verify program optimizations, (1) verify the correctness of the optimizations and develop a *verified tool*, e.g., this is the case of optimizations within the CompCert certified compiler [22] and a good number of optimizations that have been formally verified in Coq [25,17,30,31,12], (2) or use a translation validation approach [32,33,34,18] in which rather than verifying the tool, each of the compiled/optimized programs are formally checked to be correct using a verified checker. We argue that translation validation [32] is the most appropriate approach for verifying EVM optimizations because: (i) EVM compilers (together with their built-in optimizers) are continuously evolving to adjust to modifications in the rather new blockchain programming languages, (ii) existing EVM optimizers use external components such as SMT solvers to search for the optimized code and verifying an SMT solver would require a daunting effort, (iii) we aim at generality of our tool rather than restricting ourselves to a specific optimizer and, as already explained, the design of our checker has been done having generality and extensibility in mind, so that new optimizations can be easily incorporated. Finally, it is worth mentioning the KEVM framework [19], which in principle could be the basis for verifying optimizations as well. However, we have chosen to develop it in Coq due to its maturity.

⁴ https://github.com/costa-group/gasol-optimizer/commit/fd78e126c23f192e d6c54aea713b5c94d3c943f5

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A Rule simplifications

Fig. 1 includes rule simplifications that are applied by tools like GASOL and the Solidity compiler solc. Those rules integrated in our checker are marked with *.



Fig. 1. Simplification rules

B Supported Instructions

In Sec. 3 we have discussed a limited number of instructions that are included in the type stack_op_instr. Next we overview the full list of supported instructions. The following stack operations are supported in the checker:

ADD	MUL	NOT	SUB	DIV	SDIV	MOD
SMOD	ADDMOD	MULMOD	EXP	SIGNEXTEND	LT	GT
SLT	SGT	EQ	ISZERO	AND	OR	XOR
BYTE	SHL	SHR	SAR			

They all perform computations that depend only on values stored on the stack, and the result is pushed again to the stack. In addition, in order to increase the set of benchmarks that we can handle, the following are also supported, even though they depend on values stored in the memory/store/contract/etc, in a sound way:

SHA3	KECCAK256	ADDRESS	BALANCE	ORIGIN
CALLER	CALLVALUE	CALLDATALOAD	CALLDATASIZE	CODESIZE
GASPRICE	EXTCODESIZE	RETURNDATASIZE	EXTCODEHASH	BLOCKHASH
COINBASE	TIMESTAMP	NUMBER	DIFFICULTY	GASLIMIT
CHAINID	SELFBALANCE	BASEFEE	SLOAD	MLOAD
PC	MSIZE	GAS		

The support is done by corresponding implementations that always return 0. This is sound for now since (1) they are not used in rule simplifications, and thus they are never executed; and (2) we do not have any instruction that modifies the memory/store. Recall that when comparing symbolic states, if two calls to the same instruction are compared then we require their arguments to be equal and we do not inspect the returned value. For now, in the implementations, they are defined as stack_op_instr but this will change once other instruction categories are formalized.

C Missing code

```
Definition evm_add (args: list EVMWord) : option EVMWord :=
match args with
| [a; b] ⇒ Some (wplus a b) (* wplus is part of the bbv library *)
| _ ⇒ None
end.
Definition evm_mul (args: list EVMWord) : option EVMWord :=
match args with
| [a; b] ⇒ Some (wmult a b) (* wmult is part of the bbv library *)
| _ ⇒ None
end.
```

```
\begin{array}{l} \texttt{Definition stack_op_map_comm (ops: stack_op_map) : Prop :=} \\ \forall (instr : stack_oper_instr) (f: list EVMWord \rightarrow option EVMWord), \\ ops instr = Some (StackOp true 2 f) \rightarrow \\ \forall (a b: EVMWord), f [a; b] = f [b; a]. \\ \end{array}
```

D Smart contracts used in the experiments

1. 0x0621213b273bff05d679d9b1c68ec18cf989168f

For the experimental evaluation we have extracted all the blocks in the following 96 smart contracts from the Ethereum blockchain. We identify the smart contracts by their addresses.

- $2. \ 0x0 edc5 ac3 da3 df2 e4643 aca1 a777 ca9 eb6656117 a$ 3. 0x0f066b014adb057cdfc6c587965fbaa14151dfa5 $4. \ 0x100739d55a4e8361dcca7e872426c4b6aadeb201$ 5. 0x16c19aaae850bb0282b38686fb071fe37edecf1f 6. 0x16d1884381d94b372e6020a28bf41bbabe8c1f26 7. 0x1c52b09ccddd1b6999400b038c7e0680eaf03dcd 8. 0x1Ee8923Db533Ecb7A4650cCc8829D6F114D718f9 9. 0x269028c988db0e6786de1f4ff66af7c033d0f6c8 10. 0x2ccc239e97d01ad4c39a323327bc1a1f4cb43076 11. 0x2e12AE85aF4121156F62ad4D059415C746fe615c 12. 0x34662bf3ad9b3a70ea5b46ad81f4e9cab4d89a7f13. 0x359651fb6636cb650Fa47F11C9D35533BbFF8158 $14. \ 0x3948b7b6b8812439ddcbc8fa42cac8e213191792$ $15. \ 0x3 a e 30 b b 991 b e 0 d 54 f d d f e d c 7 d 6556 e 20 d a a 97 c 71$ 16. 0x3e456b66425f02bfe3896c1cc3b8513ff660b4bf 17. 0x4152e8133d79279881013789100246a907884b9e18. 0x4226cac9567e991f956f644b656ce4aa0599c63e 19. 0x44f8217a9dbb45ef2491da6aa18826bd6ded7847 20. 0x450f184242894d068a71d3abfa296a73df1e192c $21. \ 0x4757388aa7e3490106092bde16c23e2858c7d405$ 22. 0x47B51F81E03fB068d776CcB78b08F59e5256B944 23. 0x49173F2BF921Ce4124A8C6aBad3c5875Ff40D951 24. 0x49566 ab7 ef0 d4 da06 a 3117 e9 e4 fb3 e9947 aba f9625. 0x4d2d88d73ab4062d61b1eb68b5808b9176cef27126. 0x4d37D0aB328e1D449Ea8CFc3b0B7364B398c41E0 27. 0x4E4bd1f64232450bEa37c4CB76D4b4cda3d46DAa 28. 0x4f73c17195d0f77c1fc4175345b9251a9fb21849 29. 0x4f89a0d9d868a39ec7024828dcaaae140a1a7ff3 30. 0x5036f390F631f66284253864aE351B0297E32f03 $31. \ 0x50b6c438f108b5c0145142f54d538e704c55995b$ 32. 0x54adf7604d25ac9476fc467e93dfdb29af1077ee 33. 0x58760b75093a8462eb2eab2c5769ba5c0b18fa68 34. 0x5d2fdd14e44b090f2eef03c715d414039f86d7bd $35. \ 0x60e600a4d09297f9e9bb6eb90373f48e7830e69c$
- 36. 0x6365303A5E1C1327b36bDa8C22440be94eCCbcA1

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37. 0x64b88f10faf1603b70fb7370a00c43369f329515 38. 0x697720ee431148a586a546551de6c4d575e4d8d0 $39. \ 0x6cd5a65e85c9603df74d4311d76145820556548a$ 40. 0x6e53a6441b24cb773adcc6e6f9d43e956e7c9a6e $41. \ 0x70001 ba1 ba4 d85739 e7 b6 a7 c646 b8 aba5 ed6 c888$ 42. 0x702197775Ab2B462Af51Ba11b38d103AaA0bb443 43. 0x72BD2930663b30dBA3cd362bc1f8C2251E24C73A 44. 0x766a339751Df1705364D961b4f7f87309F210355 45. 0x7a741d7ff3da76d722fa4a37455f099efd0ef168 $46. \ 0x7bd251d43d8ee259acde7788ec93b7f3d6626dd2$ 47. 0x7dDA9F944c3Daf27fbe3B8f27EC5f14FE3fa94BF 48. 0x7F197F94cA6e57Fd983cE0fa29710cfA3b842bf8 49. 0x89872650fa1a391f58b4e144222bb02e44db7e3b 50. 0x8EfbD976709c2bD46bdaf0c3Db83E875F1451BAE 51. 0x8f093895cd4c54eab897c6377e1bf85fe5b4e948 $52. \ 0x8f3b62dd6a9bf905516f433c214753934b337e05$ 53. 0x90f24a2432a8b2e87b5a2026855181317890d129 54. 0x949205a8e58bd1e5eb043c6379d1e7564a85039a55. 0x94a79038D97e22AC47C9Aa41624f948BDd7ac27D 56. 0x99E2C293A8A6c3dAE6A591CEA322D0c3Cd231B2C 57. 0xa403f555e419e56F49ba90022f7E7d0d3e86522D $58. \ 0xa7b30042c7e798d0be8e466bf879388acddc526f$ 59. 0xaa30979b30523bff7ca2ba434d126d15ad5b0905 60. 0xAa7B19b68a1da16f272564e74b0e99f969c4DF6a 61. 0xAbF18841Dca279a030bd9A9122F4460Da57ad547 62. 0xAbf52Fc6e5C0e6E44Daac7C6ca79498302D9B0CA 63. 0xb4e2ebaf639fd03aebe85bd0960b49ade9879b0f 64. 0xb4feb1f99fc9e2688729fc899e1ee3631bbebded65. 0xb5615b9799427280cbc34a33233ef59b6409a711 66. 0xb595e208833164d43a08ce529acc2b803d33c30e 67. 0xb6105c0fa743290f94da9bf304ac45c19f4b2851 68. 0xb8ec5b27de7d971d74e8531baa27853cffdfae1d 69. 0xbb4ed94d1f743a8bba6318c821a7e9cf1d632c96 70. 0xbdafddc47d1cbac27f80a918908922aa6bf4b5bc 71. 0xbdf9d5752ec89a3b7c3b7ffe31f5bd565016c221 72. 0xc581bced4dedab50e8bfdcc67b2a36b92e013d78 73. 0xcbdbc7c264c1abff6646bdd5ba13f1664823b0cd 74. 0xcc07e50a953c8c61f5dc077ed171e46210e9783e75. 0xcCFFC73347B05cBDCCe7c3de2a0AdEcDAa8AEf51 76. 0xcD097cdB473286a10Cf19CbA9597E4400e8B6943 $77. \ 0xcfc 131c 7810f 9f 7ec 859 bd 3dd 020 bd b4 bb 06 a5 a 8$ 78. 0xd2947e1e2ea5c4cd14aaa2b7492549129b087daa 79. 0xda44b167ca409b7fc51ccbd6ef3338b8e19999a8 80. 0xde1972989f633f10b6e6dc581b785c4618aa8490 81. 0xdfd5235a6d3e184ba27307d7d21ae9b425ff4e6d 82. 0xe45D283123607B7D97856d49C965faa40542BA94

- $83. \ 0 x e7 a 2241 e92 c7 b7299452 e63 d53 a f 6692 df cd 0367$
- $84. \ 0xe8d2f4b9edbb0244167339c3a8daa6d82d959e72$
- $85. \ 0x Ead C2a 6ff f 036 C 12e 62 A 74392 d 4c 6 C A 77 A 5 Ea 007$
- 86. 0xeb453a070c20e79ff148e0506bd02c30b577af43
- 87. 0xEf78B55bD7bC090F809535f3B32Bcf1E050815df
- 88. 0xF0B0ccED14b2d1D47C351F5Bc0B33AA79470997e
- 89. 0xf1cb8f9738adff8c280d6eae8e2285a839b79f80
- 90. 0xF2281cA8693B1d35D7a73700909ec8535A57156D
- 91. 0xf508bda527d4ef9db712eb0100f1cd8f573bbe88
- $92. \ 0xf 66 ff 968773 e 45 dad 3 e 1 ac 13 ff b b 63 fae 0 e b 1624$
- $93. \ 0xf7a84 edAc5539 b75 AFaaA04 f1103 dBf9 Db4 b09 c6$
- 94. 0xfa1c9bf3de714059b3c019facdcaef785cab098e
- 95. 0xfc21969625ae8933e85b49df3cc28aa7092fcfc7
- $96. \ 0xfeff 9661617 cbba5a2 ed3a69000 f4bf1 e266 be71$