Implementations of Herbrand's Theorem Exercises

1 Gilmore's Method

Exercise 1. Apply Gilmore's method to the following formulæ, and try to decide their satisfiability:

1. $(\neg p \lor q) \land \neg q \land p$ 2. $(p \lor q) \land (r \lor q) \land \neg r \land \neg q$ 3. $p \land q \land r$ 4. $(p \lor q) \land (\neg p \lor q) \land r$ 5. $(p \lor q) \land \neg q$ 6. $(p \lor q) \land (\neg p \lor q) \land (\neg r \lor \neg q) \land (r \lor \neg q)$

2 The method of Davis-Putnam

Exercise 2. Apply the method of Davis-Putnam to the six formulæ above, and try to verify if their are satisfiable.

Exercise 3. Consider the following chemical reactions

$$\begin{array}{c} MgO + H_2 \rightarrow Mg + H_2O \\ C + O_2 \rightarrow CO_2 \\ CO_2 + H_2O \rightarrow H_2CO_3 \end{array}$$

and suppose there is a sufficient amount of MgO, H_2 , O_2 , and C. Show, by means of the method of Davis-Putnam, that we can make H_2CO_3 .

Exercise 4. Prove, by Davis-Putnam, that the following formula is valid.

$$(((q \to p) \land (p \to q)) \to (\neg q \land \neg r)) \lor (((r \to p) \land (q \to s)) \to ((p \to r) \to (r \land s)))$$

3 The Resolution method of Robinson

Exercise 5. Prove the following set of clauses is unsatisfiable by ground resolution:

$$\{p \lor q \lor r, \neg p \lor r, \neg q, \neg r\}$$

Exercise 6. For the set $S = \{p \lor q, \neg q \lor r, \neg p \lor q, \neg r\}$ derive an empty clause from S by ground resolution.