Computational Logic

Recall of First-Order Logic

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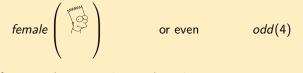
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Names, individuals, true notions

Names are only there to denote things

• there is no problem in taking



to be true if we are always consistent about it

• never forget that you are dealing with symbols!

Types in first-order logic!

- a function of arity *n* will *never* have arity $m \neq n$ in the same system
- a predicate of arity n will *never* have arity $m \neq n$ in the same system
- a function is *not* a predicate (*)
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Equality

- the predicate = /2 is special in the sense that its meaning is often given for granted in a formal system (i.e., being equal means being the same element of the domain)
- anyway, we could choose to redefine it!

Formalization, truth and well-formedness

- a formula can be well-formed without being true
- a formula can be a correct formalization of a sentence without being true or even (to our intuition) reasonable
- don't worry: sometimes a big formula may be needed to express a small sentence, or the other way around

Interpretations

When we want to prove that a formula is not valid, we need *one* interpretation which makes it false

• you can choose anything you want as D and I

• it's ok if we take *that* function s/1 to map 45 to $\frac{1}{2}$!

Implication

- that the left-hand side of an implication is false is enough to say that the implication is true
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Implication, cont.

● all red apples are good ~~

$$\forall x ((apple(x) \land red(x)) \rightarrow good(x))$$

- $\bullet\,$ there are apples which are red and good $\rightsquigarrow\,$ NO
- $\bullet\,$ all apples are red and good $\rightsquigarrow\,$ NO
- \bullet there are no apples which are red but not good \rightsquigarrow YES
- the set of good apples is a superset of the set of red apples \rightsquigarrow YES
- whenever an apple is not good, it cannot be red → YES
- $\bullet\,$ whenever an apple is not good, it must be red $\rightsquigarrow\,$ NO
- there exists a yellow apple which is bad ~~

 $\exists x (apple(x) \land yellow(x) \land bad(x))$

- $\bullet\,$ whenever an apple is yellow, it is bad $\rightsquigarrow\,$ NO
- $\bullet\,$ there is at least an object which is bad and yellow, and is an apple $\rightsquigarrow\,$ YES
- every time an apple is good, it is not yellow → NO